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VIEW FACTORS FOR TOROIDS AND THEIR PARTS

by Norman T. Grier and Ralph D. Sommers

Lewis Research Center
Cleveland, Ohio

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ABSTRACT

Various radiation view factors for a toroid have been calculated using a high-speed digital computer. The results are presented in graphs and tables. The view factors included are (1) between differential elements and bands on a toroid, (2) between differential- and finite-sized elements, (3) between finite-sized elements and the whole toroid, and (4) between a toroid and itself. The range of toroidal parameters used was such that all possible toroids are included in the calculations. Orthogonal toroidal coordinates were used in the computer calculation. Toroidal coordinates and their use in writing the view factor equations are discussed.

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SUMMARY

Various radiation view factors for a toroid have been calculated using a high-speed digital computer. The results are presented in graphs and tables. The view factors included are (1) between differential elements and bands on a toroid, (2) between differential- and finite-sized elements, (3) between finite-sized elements and the whole toroid, and (4) between a toroid and itself. The range of toroidal parameters used was such that all possible toroids are included in the calculations. Orthogonal toroidal coordinates were used in the computer calculation. Toroidal coordinates and their use in writing the view factor equations are discussed.

INTRODUCTION

One of the factors in the design of spacecraft components is the thermal environment which the components will experience on a spacecraft. Component temperatures will be determined by the thermal environment and the components will be chosen so as to survive those temperatures. Thus, knowledge of the temperatures that components will attain on a spacecraft is essential for designing those components.

Component temperatures are determined, in part, by the energy exchange between the components and other portions of the spacecraft. This energy exchange could occur in a combination of conductive, convective, and radiative processes. But the relative absence of a surrounding gas in space removes the outer convective heat exchange. Radiation and conduction remain as the modes of heat exchange between various sections of the spacecraft.

The radiative exchange between sections of a spacecraft can be characterized by geometric factors (i.e., factors determined by the spatial geometry of the spacecraft) variously called Radiant Interchange Factors, Radiation Configuration Factors, Angle Factors, Radiation Shape Factors, and View Factors. This report will use the term View Factor.

Many authors have considered view factors for basic shapes such as planes, spheres,

cylinders, and cones. An extensive treatment and compilation of radiation view factors is given in reference 1. To our knowledge, there has been no treatment of the view factors for the toroidal shape which has been proposed for future manned space stations.

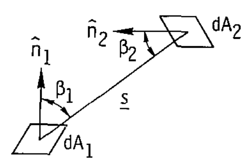
This report, therefore, deals with the toroidal shape and the view factors between various portions of a toroid. All of these are considered in a toroidal coordinate system.

VIEW FACTOR

The view factor between two areas is that fraction of radiation emitted by one which is intercepted by the other. The basic view factor equation is one which expresses the view factor between two differential elements of area. That equation is (ref. 2)

$$dF_{dA_1-dA_2} = \frac{|\cos \beta_1 \cos \beta_2| dA_2}{\pi |\underline{S}|^2} \tag{1}$$

where (sketch a) β_1 is the angle between the normal to dA_1 and $\underline{S}$, β_2 is the angle between the normal to dA_2 and $\underline{S}$, $\hat{n}_1$ and $\hat{n}_2$ are the unit normals to dA_1 and dA_2, respectively, and $\underline{S}$ is the vector distance from dA_1 to dA_2.



(a) View factor geometry.

In order to determine the view factor between portions of a toroid, this basic differential view factor equation must be integrated. The integration is done such that the differential elements of area are constrained to lie on the surface of the toroid.

Since toroids are one of the sets of coordinate surfaces in the orthogonal toroidal coordinate system, this report derives and analyzes the view factor equations using the toroidal coordinate system.

TOROIDAL COORDINATES

Toroidal coordinates represent a system of orthogonal sets of toroids, spheres, and

planes. Each toroid is defined by a coordinate μ, each sphere by a coordinate θ, and each plane by φ.

Morse and Feshbach (ref. 3) give the equations for the Cartesian coordinates x, y, and z in terms of the toroidal coordinates μ, θ, and φ.

$$\left.\begin{aligned} x &= \frac{a \cos \varphi \sinh \mu}{\cosh \mu - \cos \theta} \\ y &= \frac{a \sin \varphi \sinh \mu}{\cosh \mu - \cos \theta} \\ z &= \frac{a \sin \theta}{\cosh \mu - \cos \theta} \end{aligned}\right\} \tag{2}$$

The parameter "a" fixes the physical scale of the coordinates. With equation (2) the following can be derived and are presented in this section: (1) the equations for the toroidal coordinate surfaces, (2) the unit vectors of the toroidal coordinate system in terms of the Cartesian unit vectors, (3) the Cartesian unit vectors in terms of the toroidal unit vectors, and (4) the elements of areas in the toroidal system.

Toroidal Coordinate Surfaces

Toroids. - Each toroid is defined by a coordinate μ. The toroids encircle the z-axis. Each toroid is generated by revolving a small circle of radius r about the z-axis. The center of the small circle is a distance R from the z-axis and is always in the x, y-plane. The plane of the small circle is perpendicular to the x, y-plane.

The associated μ coordinate is

$$\operatorname{sech} \mu = \frac{r}{R} \tag{3}$$

The equation for the toroids is

$$(w - a \coth \mu)^2 + z^2 = a^2 \operatorname{csch}^2 \mu \tag{4}$$

where $w^2 = x^2 + y^2$. Equation (4) describes a toroid with small radius $r = a \operatorname{csch} \mu$ and large radius $R = a \coth \mu$.

Spheres. - The sphere centers extend up and down the z-axis. If a sphere's center is height H_c above the x, y-plane and if its radius is R_s, then the associated θ coordinate is

$$\cos \theta = \frac{H_c}{R_s} \tag{5}$$

The equation for the spheres is

$$x^2 + y^2 + (z - a \cot \theta)^2 = a^2 \csc^2\theta \tag{6}$$

Equation (6) describes a sphere with radius $R_s = a \csc \theta$ with its center at $z = a \cot \theta$.

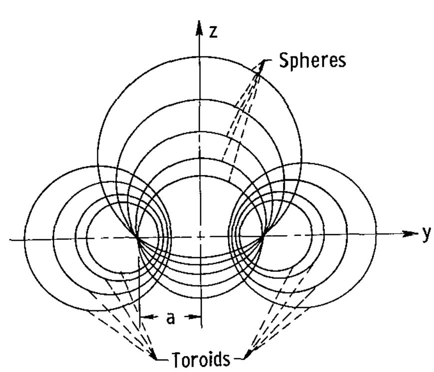
Planes. - The planes are all perpendicular to the x, y-plane and they are all joined along the z-axis (i.e., the z-axis is as though it were the spine of a book and the planes were the opened pages).

The x, z-plane is considered as the starting plane. If there is a second plane rotated an angle φ away from the x, z-plane, then φ is the associated φ-coordinate of the second plane. (The x-positive half of the x, z-plane has a φ-coordinate of zero.)

The equation for the planes is

$$\frac{y}{x} = \tan \varphi \tag{7}$$

Sketch b shows a cross section of the toroids and spheres in the two planes $\varphi = 90^{\circ}$ and 270°. Notice that all of the spheres pass through the circle $x^2 + y^2 = a^2$ in the x, y-plane.



(b) Toroids and spheres in cross section.

Toroidal Unit Vectors

Let $\hat{a}_\mu$, $\hat{a}_\theta$, $\hat{a}_\varphi$ be the unit vectors in the toroidal coordinate system. Then (ref. 3)

$$\hat{a}_n = \gamma_{n1}\hat{i} + \gamma_{n2}\hat{j} + \gamma_{n3}\hat{k} \qquad (n = \mu, \theta, \varphi) \tag{8}$$

where

$$\left.\begin{aligned} \gamma_{n1} &= \frac{1}{h_n}\frac{\partial x}{\partial \xi_n} \\ \gamma_{n2} &= \frac{1}{h_n}\frac{\partial y}{\partial \xi_n} \\ \gamma_{n3} &= \frac{1}{h_n}\frac{\partial z}{\partial \xi_n} \end{aligned}\right\} \tag{9}$$

and

$$\left.\begin{aligned} \xi_\mu &= \cosh\mu \\ \xi_\theta &= \cos\theta \\ \xi_\varphi &= \cos\varphi \end{aligned}\right\} \tag{10a}$$

$$\left.\begin{aligned} h_\mu &= \frac{a}{(\cosh\mu - \cos\theta)\sinh\mu} \\ h_\theta &= \frac{a}{\sin\theta(\cosh\mu - \cos\theta)} \\ h_\varphi &= \frac{a\cdot\sinh\mu}{\sin\varphi(\cosh\mu - \cos\theta)} \end{aligned}\right\} \tag{10b}$$

Using equations (2), (9), and (10) and writing equation (8) as a matrix we have

$$\begin{pmatrix} \hat{a}_\mu \\ \hat{a}_\theta \\ \hat{a}_\varphi \end{pmatrix} = \begin{pmatrix} \dfrac{\cos\varphi(1 - \cosh\mu\cos\theta)}{\cosh\mu - \cos\theta} & \dfrac{\sin\varphi(1 - \cosh\mu\cos\theta)}{\cosh\mu - \cos\theta} & -\dfrac{\sin\theta\sinh\mu}{\cosh\mu - \cos\theta} \\ \dfrac{\cos\varphi\sin\theta\sinh\mu}{\cosh\mu - \cos\theta} & \dfrac{\sinh\mu\sin\theta\sin\varphi}{\cosh\mu - \cos\theta} & \dfrac{1 - \cosh\mu\cos\theta}{\cosh\mu - \cos\theta} \\ \sin\varphi & -\cos\varphi & 0 \end{pmatrix} \begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} \tag{11}$$

Since the toroidal system is an orthogonal system, we can find $\hat{i}$, $\hat{j}$, and $\hat{k}$, in terms of $\hat{a}_\mu$, $\hat{a}_\theta$, and $\hat{a}_\varphi$ by using the transpose matrix of equation (11)

$$\begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = \begin{pmatrix} \dfrac{\cos\varphi(1 - \cosh\mu\cos\theta)}{\cosh\mu - \cos\theta} & \dfrac{(\cos\varphi\sinh\mu\sin\theta)}{\cosh\mu - \cos\theta} & \sin\varphi \\ \dfrac{\sin\varphi(1 - \cosh\mu\cos\theta)}{\cosh\mu - \cos\theta} & \dfrac{\sinh\mu\sin\theta\sin\varphi}{\cosh\mu - \cos\theta} & -\cos\varphi \\ \dfrac{-\sinh\mu\sin\theta}{\cosh\mu - \cos\theta} & \dfrac{1 - \cosh\mu\cos\theta}{\cosh\mu - \cos\theta} & 0 \end{pmatrix} \begin{pmatrix} \hat{a}_\mu \\ \hat{a}_\theta \\ \hat{a}_\varphi \end{pmatrix} \tag{12}$$

Toroidal Area Element

The vector elements of area are

$$\vec{dA} = \hat{a}_\mu h_\theta h_\varphi \, d\xi_\theta \, d\xi_\varphi \qquad \mu, \theta, \varphi \ \text{cyclic} \tag{13}$$

where the h and ξ quantities are given by equation (10). The surface of the toroid is defined by μ constant. The element of area of the toroid is, therefore,

$$\vec{dA} = \hat{a}_\mu \frac{a^2 \sinh\mu}{(\cosh\mu - \cos\theta)^2} \, d\theta \, d\varphi \tag{14}$$

From equation (14), the element of area is largest on the toroid when $\theta = 0$, and smallest when $\theta = \pi$. The points $\theta = 0$ correspond to points on the outside equator of the toroid; $\theta = \pi$ correspond to points on the inside equator. Thus, the element of area increases as it moves from the inside equator to the outside equator of the toroid.

VIEW FACTOR IN TOROIDAL COORDINATES

The toroids are defined by μ being constant. For computational purposes the quantity ρ is used to specify the toroids rather than μ.

The Quantity ρ

The quantity ρ is defined as

$$\rho = \frac{\text{small radius of toroid}}{\text{large radius of toroid}} = \frac{r}{R} \tag{15}$$

The two radii are given by

$$\left.\begin{aligned} r &= a \operatorname{csch} \mu \\ R &= a \coth \mu \end{aligned}\right\} \tag{16}$$

So that

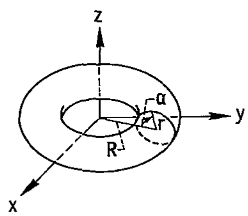
$$\rho = \frac{1}{\cosh \mu} = \operatorname{sech} \mu \tag{17}$$

The equations of the previous sections may be written in terms of ρ by using the relations

$$\left.\begin{aligned} \cosh \mu &= \frac{1}{\rho} \\ \sinh \mu &= \frac{1}{\rho}\sqrt{1 - \rho^2} \end{aligned}\right\} \tag{18}$$

The Variable α

For any given point on a toroid, there are simple ways of finding the point's θ coordinate. One way is through the variable α. This variable is defined as the angle that the small radius r of the toroid makes with the large radius R in the x, y-plane. This is depicted in sketch c below.



(c) Variable α.

With sketch c we see that

$$\cos\alpha = \frac{\text{projection of } r \text{ onto x, y-plane}}{r} = \frac{R - \sqrt{x^2 + y^2}}{r} \tag{19}$$

Then using equations (2), (16), (17), and (18) we find

$$\cos\alpha = \frac{\rho - \cos\theta}{1 - \rho\cos\theta} \tag{20}$$

and

$$\cos\theta = \frac{\rho - \cos\alpha}{1 - \rho\cos\alpha} \tag{21}$$

We refer to the points for which $\alpha = \pi/2$ as being along the top center of the toroid. We see from equation (21) that the θ coordinates for these points are given by $\cos\theta = \rho$. This relation is used later in determining the limits of integration over θ of the view factor equations.

Element to Element View Factor - $F_{dA_1-dA_2}$

The view factor from the element of area dA_1 to the element of area dA_2 on the toroid can be found from equation (1).

The unit vectors normal to the surface of the toroid are just the $\hat{a}_\mu$. With sketch a we can write

$$\cos\beta_1 = \frac{|\underline{S} \cdot \hat{a}_\mu|}{|\underline{S}|} \tag{22a}$$

where $\hat{a}_\mu$ is evaluated at $(\rho, \theta_1, \varphi_1)$.

$$\cos \beta_2 = \frac{|\underline{S} \cdot \hat{a}_\mu|}{|\underline{S}|} \tag{22b}$$

where $\hat{a}_\mu$ is evaluated at $(\rho, \theta_2, \varphi_2)$. From equation (11) and using equation (18)

$$\hat{a}_\mu = \hat{i} \cos \varphi \left(\frac{\rho - \cos \theta}{1 - \rho \cos \theta}\right) + \hat{j} \sin \varphi \left(\frac{\rho - \cos \theta}{1 - \rho \cos \theta}\right) - \hat{k} \frac{\sqrt{1 - \rho^2} \sin \theta}{1 - \rho \cos \theta} \tag{23}$$

Before equation (22) and the equation for the view factor can be evaluated, an equation for $\underline{S}$ must be derived. This is done by first writing $\underline{S}$ in Cartesian coordinates.

Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the Cartesian coordinates of the points at dA_1 and dA_2 on the toroid. Then we have

$$\underline{S} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \tag{24a}$$

$$\underline{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k} \tag{24b}$$

Then from equations (2) and (18) we find

$$\left.\begin{aligned} x &= a \cos \varphi \frac{\sqrt{1 - \rho^2}}{1 - \rho \cos \theta} \\ y &= \frac{a \sin \varphi \sqrt{1 - \rho^2}}{1 - \rho \cos \theta} \\ z &= \frac{a \rho \sin \theta}{1 - \rho \cos \theta} \end{aligned}\right\} \tag{25}$$

Therefore:

$$S_x = \frac{a \sqrt{1 - \rho^2}}{(1 - \rho \cos \theta_2)(1 - \rho \cos \theta_1)} \left[\cos \varphi_2(1 - \rho \cos \theta_1) - \cos \varphi_1(1 - \rho \cos \theta_2)\right] \tag{26a}$$

$$S_y = \frac{a \sqrt{1 - \rho^2}}{(1 - \rho \cos \theta_2)(1 - \rho \cos \theta_1)} \left[\sin \varphi_2(1 - \rho \cos \theta_1) - \sin \varphi_1(1 - \rho \cos \theta_2)\right] \tag{26b}$$

$$S_z = \frac{\rho a}{(1 - \rho \cos \theta_2)(1 - \rho \cos \theta_1)} \left[\sin \theta_2 (1 - \rho \cos \theta_1) - \sin \theta_1 (1 - \rho \cos \theta_2) \right] \tag{26c}$$

Now

$$|\underline{S}|^2 = S_x^2 + S_y^2 + S_z^2 \tag{27}$$

So substituting the equations (26) into (27) we obtain

$$|\underline{S}|^2 = \frac{2a^2 \left\{ 1 - \cos(\varphi_2 - \varphi_1) + \rho^2 \left[\cos(\varphi_2 - \varphi_1) - \cos(\theta_2 - \theta_1) \right] \right\}}{(1 - \rho \cos \theta_1)(1 - \rho \cos \theta_2)} \tag{28}$$

And from equations (22) to (24) and (26) we find

$$\cos \beta_1 = \left| \frac{a \sqrt{1 - \rho^2}}{|S|(1 - \rho \cos \theta_2)(1 - \rho \cos \theta_1)} \left\{ \cos \theta_1 \left[1 - \cos(\varphi_2 - \varphi_1) \right] + \rho \left[\cos(\varphi_2 - \varphi_1) - \cos(\theta_2 - \theta_1) \right] \right\} \right| \tag{29a}$$

$$\cos \beta_2 = \left| \frac{a \sqrt{1 - \rho^2}}{|S|(1 - \rho \cos \theta_1)(1 - \rho \cos \theta_2)} \left\{ \cos \theta_2 \left[1 - \cos(\varphi_2 - \varphi_1) \right] + \rho \left[\cos(\varphi_2 - \varphi_1) - \cos(\theta_2 - \theta_1) \right] \right\} \right| \tag{29b}$$

Now using equations (1), (28), and (14) we have for the view factor between the elements dA_1 and dA_2

$$dF_{dA_1 - dA_2} = \frac{\rho \sqrt{1 - \rho^2}\, (1 - \rho \cos \theta_1) \cos \beta_1 \cos \beta_2 \, d\Phi \, d\theta_2}{2\pi (1 - \rho \cos \theta_2) \left\{ 1 - \cos \varphi + \rho^2 \left[\cos \varphi - \cos(\theta_2 - \theta_1) \right] \right\}} \tag{30}$$

where $\Phi = (\varphi_2 - \varphi_1)$ and $\cos \beta_1$ and $\cos \beta_2$ are given by equations (29a) and (29b).

Element to Differential φ-Segment View Factor, $dF_{dA_1-d\varphi}$

If equation (30) is integrated over the complete range of θ_2, we obtain the view factor between the differential element dA_1 and a hoop-shaped differential φ-segment. The differential width to the segment is $d\varphi$.

Even though equation (30) is being integrated over the entire range of θ_2 (0 to 2π), the integrand becomes zero beyond certain values of θ_2. This occurs because of tangency of $\underline{S}$ at either dA_1 or dA_2. These limits will be discussed in the next section.

The view factor from dA_1 to a differential φ-segment is

$$F_{dA_1-d\varphi} = \int_{\theta_{2min}}^{\theta_{2max}} dF_{dA_1-dA_2} \tag{31}$$

The Limits θ_{2min} and θ_{2max}

The limits on θ_2 in equation (31) are set by the tangency of $\underline{S}$ (the line of sight between dA_1 and dA_2) at either dA_1 or dA_2. If the tangency at dA_1 is the limiting condition, we set $\cos \beta_1 = 0$ and solve for the θ_2 limit. If tangency at dA_2 is the limiting condition, we let $\cos \beta_2 = 0$ and solve for the θ_2 limit.

Applying these two conditions results in

Tangency at dA_1 ($\cos \beta_1 = 0$)

$$\cos \theta_2 = \left[\frac{\cos \theta_1}{\rho} + \left(1 - \frac{\cos \theta_1}{\rho}\right)\cos \varphi\right]\left\{\cos \theta_1 \pm \sin \theta_1 \sqrt{\frac{1}{\left[\frac{\cos \theta_1}{\rho} + \left(1 - \frac{\cos \theta_1}{\rho}\right)\cos \Phi\right]^2} - 1}\right\} \tag{32}$$

Tangency at dA_2 ($\cos \beta_2 = 0$)

$$\cos\theta_2 = \frac{-\rho\left[\cos\Phi \pm \sin\theta_1\sqrt{1 + \frac{\rho^2\left(\sin^2\theta_1 - \cos^2\Phi\right)}{\left(1 - \cos\Phi - \rho\cos\theta_1\right)^2}}\,\right]}{1 - \cos\Phi - \rho\cos\theta_1 + \frac{\rho^2\sin^2\theta_1}{1 - \cos\Phi - \rho\cos\theta_1}} \tag{33}$$

Which of these two conditions is applied depends upon the value of θ_1. In addition, each of equations (32) and (33) represents two solutions. The proper choice between these two solutions also depends upon θ_1.

Toroid Symmetry - Limits on the Range of θ_1

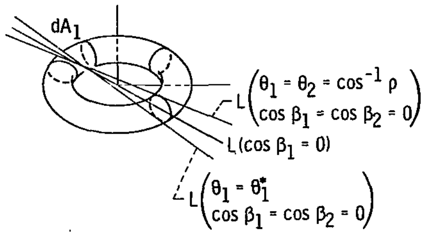
The symmetry of the toroid is such that elements on the top portion of the toroid $(0 \leq \theta_1 \leq \pi)$ "see" the rest of the toroid in exactly the same way as do elements on the bottom part of the toroid $(\pi \leq \theta_1 \leq 2\pi)$. For this reason we need only to consider θ_1 in the range $0 \leq \theta_1 \leq \pi$. The range of θ_1 is made even smaller, however, since elements on the top outside portion of the toroid $(0 \leq \theta \leq \arccos\rho)$ cannot see any other element on the toroid. Therefore, we consider θ_1 only in the range $\arccos\rho \leq \theta_1 \leq \pi$.

The Limit θ_{2min}

Because of the restricted range of θ_1, the minimum value of θ_2 will always correspond to $\underline{S}$ being tangent at dA_2. This follows since θ_{2min} will also always be on the top portion of the toroid. Therefore, θ_{2min} will always be given by one of the solutions of equation (33). That solution will be the one for which $\arccos\rho \leq \theta_{2min} \leq \theta_1$.

The Limit θ_{2max}

Let L be a line drawn tangent to dA_1. The line L goes to dA_2 which is located at some position Φ (sketch d). The value of Φ is to remain constant in the following discussion.



(d) Change of line-of-sight between dA_1 and dA_2.

Beginning with dA_1 located at θ_1 = arc cos ρ (the top center of the toroid), we let θ_1 increase while holding L tangent to dA_1. As θ_1 increases, line L will intersect the toroid at increasing values of θ_2. These values of θ_2 are the maximum values of θ_2 that are within the line-of-sight of dA_1. And for this range of θ_1 equation (32) ($\cos \beta_1 = 0$) is to be used to find θ_{2max}.

As we continue to increase θ_1 with L tangent at θ_1, L will at some point stop intersecting the toroid. Just before this happens, L will become tangent at dA_2. Both $\cos \beta_1$ and $\cos \beta_2$ are now zero. Call the value of θ_1 for which this happens θ_1^*.

If we increase θ_1 beyond θ_1^*, L leaves the toroid and does not intersect at any dA_2. From θ_1^* on, then, the limit θ_{2max} is determined by line-of-sight tangency at dA_2. For values of θ_1 greater than θ_1^* equation (33) is to be used to find θ_{2max}.

The point θ_1^* that marks the change from using equation (32) to using equation (33) is found by equating equations (29a) and (29b) to zero ($\cos \beta_1 = \cos \beta_2 = 0$). We then solve the set for θ_1^*. Doing this we have

$$(\cos \theta_1^*)(1 - \cos \Phi) + \rho\left[\cos \Phi - \cos(\theta_2 - \theta_1^*)\right] = 0 \tag{34}$$

$$(\cos \theta_2)(1 - \cos \Phi) + \rho\left[\cos \Phi - \cos(\theta_2 - \theta_1^*)\right] = 0 \tag{35}$$

Hence,

$$\cos \theta_2 = \cos \theta_1^* \tag{36}$$

and

$$\theta_2 = \theta_1^* \tag{37a}$$

or

$$\theta_2 = 2\pi - \theta_1^* \tag{37b}$$

Substituting equation (37a) into (34) we find

$$(\rho + \cos \theta_1^*)(1 - \cos \Phi) = 0 \tag{38}$$

If $\Phi \neq 0$, then

$$\cos \theta_1^* = \rho \tag{39}$$

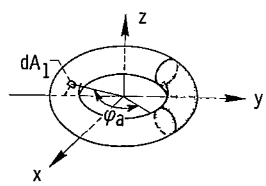
This is a special case solution of L lying horizontally across the toroid and making tangential contact along the top center points of the toroid ($\cos \theta = \rho$). More generally, we use equation (37b) to determine θ_1^*. Substituting (37b) into equation (34) we find

$$\cos \theta_1^* = \frac{1}{4\rho}\left[(1 - \cos \Phi) - \sqrt{(1 - \cos \Phi)^2 + 8\rho^2(1 + \cos \Phi)}\right] \tag{40}$$

and we remember that the range of θ_1 is $\arccos \rho \leq \theta_1 \leq \pi$.

Element to Φ-Segment View Factor, $F_{dA_1-\Phi_S}$

A Φ-segment is defined as the portion of the toroid generated when dA (eq. (14)) is integrated over the complete range of θ $(0 - 2\pi)$ and only over part of the range of Φ. Therefore, when a toroid is cut by two Φ-planes, that portion of the toroid between the two planes is the Φ-segment. This is shown in sketch e. For the view factor calculations the center of the segment is taken opposite to dA_1. The two Φ-planes are at equal angles with respect to dA_1 and the Φ-segments are thus symmetrical in Φ.



(e) Toroid Φ-segment.

The equation for the view factor from an element dA_1 to a Φ-segment symmetrically located about dA_1 is

$$F_{dA_1-\varphi_S}(\varphi_a) = \int_{\varphi_a}^{2\pi-\varphi_a} \int_{\theta_{2min}}^{\theta_{2max}} dF_{dA_1-dA_2} = 2 \int_{\varphi_a}^{\pi} \int_{\theta_{2min}}^{\theta_{2max}} dF_{dA_1-dA_2} \tag{41}$$

where $2(\pi-\varphi_a)$ is the size of the segment. Equation (41) is an integration of equation (30) over the area of the φ-segment. The θ_{2min} and θ_{2max} are the limits of the range of θ_2 in which dA_1 "sees" a dA_2.

View Factor From Element dA_1 to Different Toroidal Band, $dF_{dA_1-d\theta_2}$

If equation (30) is integrated over the complete range of Φ, we find the view factor between dA_1 and a ring-shaped differential toroidal band. The integrand, however, becomes zero at some points within that range. So, the integration is effectively done by integrating between two limits, say, Φ_{min} and Φ_{max}.

The view factor between dA_1 and a differential toroidal band is

$$dF_{dA_1-d\theta_2} = \int_{\Phi_{min}}^{\Phi_{max}} dF_{dA_1-dA_2} \tag{42}$$

The limits Φ_{min} and Φ_{max} are determined by the tangency of $\underline{S}$ at dA_1 or dA_2. Which condition is used depends on the relative locations of the element and differential toroidal band.

If $\theta_2 \leq \theta_1$, tangency of $\underline{S}$ at dA_2 sets the limits Φ_{min} and Φ_{max}. If $\theta_2 < 2\pi - \theta_1$, the limits are set by tangency of $\underline{S}$ at dA_1. These two conditions produce the next two equations.

$\theta_2 \leq \theta_1$
$(\cos \beta_2 = 0)$

$$\cos \Phi = \frac{\rho \cos(\theta_2 - \theta_1) - \cos \theta_2}{\rho - \cos \theta_2} \tag{43a}$$

$\theta_2 < 2\pi - \theta_1$
$(\cos \beta_1 = 0)$

$$\cos \Phi = \frac{\rho \cos(\theta_2 - \theta_1) - \cos \theta_1}{\rho - \cos \theta_1} \tag{43b}$$

From either of the equations (43) there are two solutions for Φ. One, Φ_{min}, lies in the range 0 to π. The other, Φ_{max}, is in the range π to 2π. In addition, the two solutions are related by

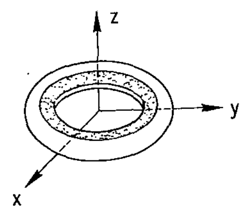
$$\Phi_{max} = (2\pi - \Phi_{min}) \tag{44}$$

Therefore, equation (42) can be written as

$$dF_{dA_1-d\theta_2} = 2 \int_{\Phi_{min}}^{\pi} dF_{dA_1-dA_2} \tag{45}$$

View Factor From Element dA_1 to Toroidal Bands

A toroidal band is defined as the surface generated when dA (eq. (14)) is integrated over the complete range of Φ and only over a part of the range of θ. A toroidal band is depicted in sketch f.



(f) Toroidal band.

The equation for the view factor from element dA_1 to a toroidal band is

$$F_{dA_1-B}(\theta_a, \theta_b) = \int_{\theta_a}^{\theta_b} \int_{\Phi_{min}(\theta_1, \theta_2)}^{\Phi_{max}(\theta_1, \theta_2)} dF_{dA_1-dA_2} \tag{46}$$

where θ_a and θ_b are the lower and upper limits of θ in the toroidal band. $dF_{dA_1-dA_2}$ is given by equation (30). (Note that the toroidal band may include dA_1.)

The limits $\Phi_{min}(\theta_1, \theta_2)$ are the limits of Φ that are within the line of sight from dA_1. This occurs when $\underline{S}$ becomes tangent at dA_2 or dA_1 as discussed in the previous section. The limits Φ_{min} and Φ_{max} are again related by

$$\Phi_{max} = 2\pi - \Phi_{min} \tag{47}$$

In the calculations the toroidal bands were chosen on the inside portion of the toroid and symmetrical about the equator. Therefore, in the calculations

$$\theta_b = 2\pi - \theta_a \tag{48}$$

By choosing the bands about the equator, the view factor may be divided into two separate view factors: (1) the view factor from element dA_1 to the upper portion of the band $F_{dA_1-B}^{(upper)}$ and (2) the view factor from element dA_1 to the lower portion of the band $F_{dA_1-B}^{(lower)}$. From equation (46) we have

$$F_{dA_1-B}^{(upper)}(\theta_a) = \int_{\theta_a}^{\pi} \int_{\Phi_{min}(\theta_1, \theta_2)}^{\Phi_{max}(\theta_1, \theta_2)} dF_{dA_1-dA_2} \tag{49a}$$

$$F_{dA_1-B}^{(lower)}(\theta_b) = \int_{\pi}^{\theta_b} \int_{\Phi_{min}(\theta_1, \theta_2)}^{\Phi_{max}(\theta_1, \theta_2)} dF_{dA_1-dA_2} \tag{49b}$$

Note that

$$F_{dA_1-B}(\theta_a, \theta_b) = F_{dA_1-B}^{(upper)}(\theta_a) + F_{dA_1-B}^{(lower)}(\theta_b) \tag{49c}$$

The advantage of dividing the view factor into these two separate view factors is that it allows the determination of the view factor from element dA_1 to any toroidal band. How this is done is shown in detail in the section EXTENSION OF VIEW FACTOR DATA.

View Factor From Element dA_1 to Toroid

The view factor from element dA_1 to the whole toroid F_{dA_1-T} is the same as the view factor from element dA_1 to the toroidal band with $\theta_a = \arccos \rho$ (i.e., $\alpha_2 = 90^o$).

View Factor Between Equatorial Toroidal Bands

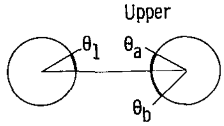
If equation (46) is integrated all the way around φ_1 and only over part of the range of θ_1, the area A_1 is then a toroidal band. This integration gives the view factor between two finite toroidal bands $F_{B_1-B_2}$.

$$F_{B_1-B_2}(\theta_1) = \frac{1}{A_1}\int_{\varphi_1=0}^{2\pi}\int_{\theta_1=\theta_1}^{\pi} F_{dA_1-B}(\theta_a, \theta_b)dA_1 \tag{50}$$

$$dA_1 = \frac{a^2\rho\sqrt{1 - \rho^2}}{\left(1 - \rho \cos \theta_1\right)^2} \tag{51}$$

$$A_1 = a^2\rho\sqrt{1 - \rho^2}\int_{\varphi_1=0}^{2\pi}\int_{\theta_1}^{\pi}\frac{d\theta_1\, d\varphi_1}{\left(1 - \rho \cos \theta_1\right)^2} \tag{52}$$

As in the preceding case, it is useful to consider the receiving bands (i.e., A_2) as starting at the inside equator of the toroid and extending in either direction. We will call this particular type of band an equatorial band (see sketch g).



(g) Equatorial bands (cross-section).

The view factors are presented in terms of a view factor to an upper band and a view factor to a lower band. The two view factors are then

$$F_{B_1-B_2}^{(upper)}(\theta_1, \theta_a) = \frac{\int_{\varphi_1=0}^{2\pi} \int_{\theta}^{\pi} F_{dA_1-B}^{(upper)} \frac{d\theta_1 \, d\varphi_1}{(1 - \rho \cos \theta_1)^2}}{\int_{\varphi_1=0}^{2\pi} \int_{\theta}^{\pi} \frac{d\theta_1 \, d\varphi}{(1 - \rho \cos \theta_1)^2}} \tag{53}$$

$$F_{B_1-B_2}^{(lower)}(\theta_1, \theta_b) = \frac{\int_{\varphi_1=0}^{2\pi} \int_{\theta_1}^{\pi} F_{dA_1-B}^{(lower)} \frac{d\theta_1 \, d\varphi_1}{(1 - \rho \cos \theta_1)^2}}{\int_{\varphi_1=0}^{2\pi} \int_{\theta_1}^{\pi} \frac{d\theta_1 \, d\varphi_1}{(1 - \rho \cos \theta_1)^2}} \tag{54}$$

The special case of the view factor between an equatorial band and its whole toroid can be obtained. This is done by letting the integration of equations (53) and (54) range over the entire toroid (i.e., $\theta_a = \arccos \rho$ and $\theta_b = 2\pi - \theta_a$). Then the view factor between an equatorial band B_1 and its toroid is

$$F_{B_1-T}(\theta_1) = F_{B_1-T}^{(upper)}(\theta_1, \arccos \rho) + F_{B_1-T}^{(lower)}(\theta_1, 2\pi-\theta_a) \tag{55}$$

The integrands of equations (53) and (54) are given by equations (49a) and (49b). Note that these equations describe view factors between equatorial toroidal bands which always have as one boundary the inside equator of the toroid! It is shown in the section EXTENSION OF VIEW FACTOR DATA how to obtain from these the factors for arbitrarily chosen bands.

EXTENSION OF VIEW FACTOR DATA

The integration of the view factor equations has been done for specially chosen area elements (e.g., the toroidal bands were symmetric about the inside toroid equator, the φ-segments were symmetric about the elemental area dA_1). But the results of these

special integrations can be applied to find view factors for general toroidal bands and φ-segments. This can be done by simple additions and subtractions of the view factors for symmetric toroidal bands and φ-segments.

The following sections describe the ways in which the data presented in this report can be extended to find view factors for several area combinations. In each case, the view factor for the general area is expressed in terms of the tabulated values of the appropriate symmetric areas.

View Factor Between Toroid and Itself

The view factor from a toroid to itself is found by letting the bands in the view factor from toroidal bands encompass the whole toroid. Mathematically this is accomplished by letting θ_1 in equations (53) and (54) range from 0 to 2π. However, points on the outside of the toroid (i.e., θ_1 in the range (2π - arc cos ρ) to arc cos ρ), cannot "see" any other points on the toroid. Therefore, for these values of θ_1, F_{dA_1-B} is zero. So we let the lower limits in equations (53) and (54) become $\theta_1 = \text{arc}\cos\rho$.

We note, however, that using $\theta_1 = \text{arc cos}\ \rho$ in equations (53) and (54) has the effect of causing the denominators of these equations to represent only the area of an inside quarter of the toroid. It is really the total area of the toroid which should be involved. So the view factor between an entire toroid and itself is

$$F_{T-T} = \frac{A_B(\text{arc cos}\ \rho)}{A_T} F_{B_1-T}(\text{arc cos}\ \rho) \tag{56}$$

where $F_{B_1-T}(\text{arc cos}\ \rho)$ is the view factor between an equatorial band extending from $\theta_1 = \text{arc cos}\ \rho$ to $\theta_1 = \pi$ and its toroid.

The equatorial band area is

$$A_B(\text{arc cos}\ \rho) = 2\pi\rho R^2\left(1 - \rho^2\right)^{3/2} \int_{\text{arc cos}\ \rho}^{\pi} \frac{d\theta}{\left(1 - \rho \cos \theta\right)^2} = \pi\rho R^2(\pi - 2\rho) \tag{57}$$

and the total area of the toroid is

$$A_T = 2\pi\rho R^2\left(1 - \rho^2\right)^{3/2} \int_0^{2\pi} \frac{d\theta}{\left(1 - \rho \cos \theta\right)^2} = 4\pi R^2 \rho \tag{58}$$

Equations (57) and (58) follow from equation (52), integrating over φ and using the relation $a^2 = R^2(1 - \rho^2)$.

View Factor From dA_1 to Any φ-Segment

The view factor from dA_1 to a φ-segment symmetrically located about dA_1 is given in equation (31). The equation for the view factor from dA_1 to <u>any</u> φ-segment is

$$F_{dA_1-\varphi_S}(\varphi_b, \varphi_c) = \int_{\varphi_b}^{\varphi_c} \int_{\theta_{2min}}^{\theta_{2max}} dF_{dA_1-dA_2} \tag{59}$$

Rewriting equation (59) as

$$F_{dA_1-\varphi_S}(\varphi_b, \varphi_c) = \int_{\varphi_b}^{\pi} \int_{\theta_{2min}}^{\theta_{2max}} dF_{dA_1-dA_2} + \int_{\pi}^{\varphi_c} \int_{\theta_{2min}}^{\theta_{2max}} dF_{dA_1-dA_2} \tag{60}$$

And comparing equation (60) with equation (31) we find

$$F_{dA_1-\varphi_S}(\varphi_b, \varphi_c) = \begin{cases} \frac{1}{2}\left[F_{dA_1-\varphi S}(\varphi_b) + F_{dA_1-\varphi_S}(\varphi_c)\right] & \text{if } \varphi_b < \pi \text{ and } \varphi_c < \pi \\ \frac{1}{2}\left[F_{dA_1-\varphi S}(\varphi_b) - F_{dA_1-\varphi_S}(\varphi_c)\right] & \text{if } \varphi_b < \pi \text{ and } \varphi_c < \pi \end{cases} \tag{61}$$

Since the view factor is invariable if we replace φ by $-\varphi$, we can restrict φ_b to the interval 0 to π.

View Factor From Any φ-Segment to Its Toroid

The view factor from a φ-segment to its toroid (which includes the φ-segment) is

$$F_{\varphi_S-T} = \frac{2a^2\rho\sqrt{1 - \rho^2}}{A_{\varphi S}} \int_{\varphi_1=\varphi_a}^{\varphi_1=\varphi_b} \int_{\arccos\rho}^{\pi} \frac{F_{dA_1-T}}{\left(1 - \rho\cos\theta_1\right)^2} d\theta_1\, d\varphi_1 \tag{62}$$

where the area of the φ-segment is

$$A_{\varphi_S} = \frac{2\pi a^2(\varphi_b - \varphi_a)\rho}{(1 - \rho^2)} \tag{63}$$

The integrand in equation (62) is not a function of φ_1 so the φ_1 integration can be performed to obtain

$$F_{\varphi_S\text{-}T} = \frac{2a^2(\varphi_b - \varphi_a)\rho\sqrt{1 - \rho^2}}{A_{\varphi S}} \int_{\arccos \rho}^{\pi} \frac{F_{dA_1\text{-}T}}{(1 - \rho \cos \theta_1)^2} d\theta_1 \tag{64}$$

Using equation (63) for $A_{\varphi S}$, equation (64) becomes

$$F_{\varphi_S\text{-}T} = \frac{(1 - \rho^2)^{3/2}}{\pi} \int_{\arccos \rho}^{\pi} \frac{F_{dA_1\text{-}T}}{(1 - \rho \cos \theta_1)^2} d\theta_1 \tag{65}$$

Notice that in equation (65) φ_a and φ_b no longer appear. This means that the view factor for any φ-segment to its toroid does not depend upon either the size or location of the φ-segment. In addition, note that equation (65) is identical to the expression for the toroid to itself view factor (obtained from eqs. (56) and (58)). Thus, not only is the view factor for any φ-segment to toroid equal to the view factor for any other φ-segment to toroid on the same toroid, but both are equal to the toroid-to-toroid view factor of that toroid.

$$F_{\varphi_S\text{-}T} \equiv F_{T\text{-}T} \tag{66}$$

View Factor From dA_1 to Any Toroidal Band

Equation (46) gives the equation for the view factor from dA_1 to any toroidal band. Writing equation (46) as

$$F_{dA\text{-}B}(\theta_a, \theta_b) = \int_{\theta_a}^{\pi} \int_{\varphi_{min}}^{\varphi_{max}} dF_{dA_1\text{-}dA_2} + \int_{\pi}^{\varphi_b} \int_{\varphi_{min}}^{\varphi_{max}} dF_{dA_1\text{-}dA_2} \tag{67}$$

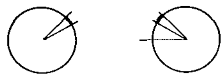
where, as before, θ_a and θ_b are the lower and upper limits in θ for the toroidal band. Using equations (49a), (49b), and (67) we have

$$F_{dA_1-B}(\theta_a, \theta_b) = \left\{ \begin{array}{ll} F^{(upper)}_{dA_1-B}(\theta_a) + F^{(lower)}_{dA_1-B} & \text{for } \theta_a < \pi \text{ and } \theta_b > \pi \\ F^{(upper)}_{dA_1-B}(\theta_a) - F^{(upper)}_{dA_1-B}(\theta_b) & \text{for } \theta_a < \pi \text{ and } \theta_b > \pi \\ -F^{(lower)}_{dA_1-B}(\theta_a) + F^{(lower)}_{dA_1-B}(\theta_b) & \text{for } \theta_a > \pi \text{ and } \theta_b > \pi \end{array} \right\} \tag{68}$$

View Factors Between General Toroidal Bands

The view factors presented in table IV are for bands which have the inside equator of the toroid as one boundary. We call such bands equatorial bands and any arbitrary toroidal band is a combination of two equatorial bands.

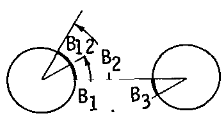
The view factors between an arbitrary toroidal band and other toroidal bands may be found (see sketch h) in terms of those between equatorial bands as follows:



(h) Toroidal bands as combination of equatorial bands.

Case I - Toroidal bands bounded by equatorial bands which are on the same side of the toroid equator

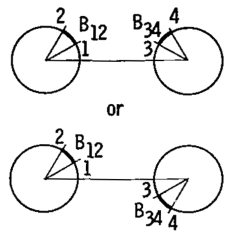
Ia: If a toroidal band B_{12} is contained between the two equatorial bands B_1 and B_2 (see sketch i), then the view factor between B_{12} and an equatorial band B_3 is



(i) Case Ia.

$$F_{B_{12}-B_3} = \frac{A_{B_2}}{A_{B_{12}}} F_{B_2-B_3} - \frac{A_{B_1}}{A_{B_{12}}} F_{B_1-B_3} \tag{69}$$

Ib: The view factor between two toroidal bands B_{12} and B_{34} (sketch j) (B_{12}



(j) Case Ib.

is contained between equatorial bands B_1 and B_2, B_{34} contained between B_3 and B_4) is given by

$$F_{B_{12}-B_{34}} = F_{B_{12}-B_4} - F_{B_{12}-B_3} \tag{70}$$

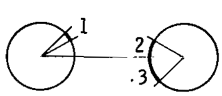
where equation (69) is used to find $F_{B_{12}-B_4}$ and $F_{B_{12}-B_3}$.

Ic: The two toroidal bands could be one and the same band. In this case, the use of equations (69) and (70) produce the self-view factor for a toroidal band.

$$F_{B_{12}-B_{12}} = \frac{A_{B_1}}{A_{B_{12}}} F_{B_1-B_1} + \frac{A_{B_2}}{A_{B_{12}}} F_{B_2-B_2} - 2 \frac{A_{B_1}}{A_{B_{12}}} F_{B_1-B_2} \tag{71}$$

Case II - Toroidal bands bounded by two equatorial bands which are on opposite sides of the toroid equator

IIa: Receiving band straddles the equator (sketch k).

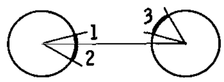


(k) Case IIa.

$$F_{B_1-B_{23}} = F_{B_1-B_2} + F_{B_1-B_3} \tag{72}$$

and $F_{B_1-B_2}$ and $F_{B_1-B_3}$ are found from the results of case Ib above.

IIb: Sending band straddles the equator (sketch l).

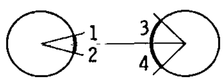


(l) Case IIb.

$$F_{B_{12}-B_3} = \frac{A_{B_1}}{A_{B_{12}}} F_{B_1-B_3} + \frac{A_{B_2}}{A_{B_{12}}} F_{B_2-B_3} \tag{73}$$

where $F_{B_1-B_3}$ and $F_{B_2-B_3}$ are found as in case Ib.

IIc: Both receiving and sending band straddle the equator (sketch m).



(m) Case IIc.

$$F_{B_{12}-B_{34}} = \frac{A_{B_1}}{A_{B_{12}}}\left(F_{B_1-B_3} + F_{B_1-B_4}\right) + \frac{A_{B_2}}{A_{B_{12}}}\left(F_{B_2-B_3} + F_{B_2-B_4}\right) \tag{74}$$

View Factor Between Any Toroidal Band and Its Toroid

To find this view factor, toroidal band to toroid, it is only necessary to apply cases IIa or IIc in the section above. We consider the toroid as a receiving area bounded by the two equatorial bands with $\alpha_2 = 90^{\mathrm{o}}$ and $\alpha_2 = -90^{\mathrm{o}}$.

RESULTS AND DISCUSSION

The calculations for the various view factors were made for ρ varying from 0.01 to 0.99. The results are presented in both tabular and graphical form. In the tables whenever the toroidal variable θ is listed, the corresponding value of α is also listed.

Figure 1 and table I present the variation of θ as a function of α for each ρ used in the calculations. The curves in figure 1 were drawn for α and θ on the upper inside quarter of the toroids. Table I covers the whole upper half of the toroid. The values of θ for the lower-half of the toroid are found by considering α to be negative, then $\theta_{-\alpha} = 2\pi - \theta_{+\alpha}$.

The view factor from element dA_1 to φ-segments are presented in table II and figure 2. The calculations were made for the element dA_1 located at α_1 in the range 0^o to 80^o at 10^o intervals. As mentioned previously, the φ-segments were chosen directly and symmetrically opposite to dA_1. Therefore, as viewed from dA_1, half of the φ-segment angle is to the right of dA_1 and the other half is to the left.

The view factor from element dA_1 to toroidal bands are presented in table III and figure 3. The elements dA_1 were chosen at values of α_1 in the range 0^o to 80^o at 10^o intervals. The toroidal bands were located symmetrically about the equator of the toroid and are thus the sum of two equal equatorial toroidal bands. The angle subtended by these bands, at the point where α_1 is measured, varied from 0^o to 180^o. Therefore, the value of the α_1 angles included in the toroidal bands varied from 0^o to $\pm 90^o$, where the plus sign corresponds to the upper half of the toroidal band and the negative sign corresponds to the lower half. In table III, values for the view factor from dA_1 to the upper half, lower half, and the total toroidal band are given for every 10^o step in α_1.

One interesting point to notice from figure 3 is that for a given α_1, as ρ increases, less and less of the lower portion of the toroidal band is within the line-of-sight from dA_1. For example, take $\alpha_1 = 20^o$, then for $\rho = 0.1$, the view factor has a value of 0.037 for the lower inside half of the toroid (fig. 3(b), $\alpha_2 = 90^o$). This is about 82 percent of its value at $\alpha_1 = 0^o$. However, for $\rho = 0.99$ (fig. 3(k)), although the lower inside of the toroid was almost completely visible at $\alpha_1 = 0^o$, it is not visible at all at $\alpha_1 = 20^o$.

Notice also in figure 3 that the curves for the upper portion of the toroid are intermixed for various α_1. For wide bands (large α_2) the view factor for small α_1 is sometimes less than the view factor for larger α_1. This arises because the element dA_1 "sees" equally the upper and lower portions of the toroid. But as dA_1 moves up the toroid, it sees more and more of the upper portion of the toroid causing an increase in the view factor. As dA_1 continues to move up the toroid (increasing α_1), the sighting angles involved cause a decrease in the view factor.

The view factors between equatorial toroidal bands are given in table IV. The bands have as one boundary the inside equator of the toroid and they extend above and below that

equator. Thus, table IV contains values for upper, lower, and total view factors.

The view factors between elements and differential φ-segments are given in figure 4. These view factors have been presented in terms of view factor per width $d\varphi$ of the differential φ-segment (i.e., the derivative of the view factor with respect of φ). Figure 4 gives the ratio $dF_{dA_1 - d\varphi}/d\varphi$ as a function of φ.

The view factors between elements and differential toroidal bands are presented in figure 5. These view factors have been plotted in terms of view factor per width $d\alpha_2$ of the differential toroidal band (i.e., the derivative of the view factor with respect to α_2). Figure 5 gives the ratio $dF_{dA_1 - d\alpha_2}/d\alpha_2$ as a function of α_2. Note that for $\rho > 0.6$, the ratio $dF_{dA_1 - d\alpha_2}/d\alpha_2$ at times has a value greater than 1. This is a result of the very rapid change of this view factor with changes in α_2 on large ρ toroids.

The toroid symmetry in φ is such that an element dA_1, located at θ_1, has a constant relation to a differential toroidal band at θ_2 regardless of the φ_1 location of dA_1. This means that the view factor between element dA_1 at θ_1 and a differential toroidal band is identical with the view factor between differential toroidal bands at θ_1 and θ_2. Mathematically this is equivalent to noting that the equations defining this view factor (eqs. (30) and (42)) are not explicitly dependent upon φ_1.

In table V the areas of equatorial toroidal bands are given. The areas are given nondimensionalized by the large radius R of the toroid. These areas are to be used to find view factors other than those tabulated.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 5, 1968,
124-09-19-03-22.

APPENDIX - SYMBOLS

A	area
dA	differential area element
$A_B(\theta_a)$	area of a toroidal band that is symmetrical about the equator (i.e., from θ_a to $(2\pi - \theta_a)$)
$A_B(\theta_a, \theta_b)$	area of a toroidal band located on a toroid from $\theta = \theta_a$ to $\theta = \theta_b$
A_T	total area of a toroid
a	physical scale parameter for toroids
$\hat{a}_\mu, \hat{a}_\theta, \hat{a}_\varphi$	toroidal coordinates unit vectors
$F_{A_1-A_2}$	view factor from area A_1 to area A_2
$F_{dA_1-dA_2}(\theta_a, \theta_b)$	view factor from differential area dA_1 to a toroidal band located on a toroid from $\theta = \theta_a$ to $\theta = \theta_b$
$F_{dA_1-B}^{(lower)}(\theta_b)$	view factor from a differential area dA_1 to a toroidal band located on a toroid from $\theta = \pi$ to $\theta = \theta_b$
$F_{dA_1-B}^{(upper)}$	view factor from a differential area dA_1 to a toroidal band located on a toroid from $\theta = \theta_a$ to $\theta = \pi$
F_{dA_1-T}	view factor from differential area dA_1 to its toroid
$F_{dA_1-\varphi_S}(\varphi_a)$	view factor from differential area dA_1 to a symmetrically located φ-segment about dA_1. The φ-segment goes from φ_a to $2\pi - \varphi_a$
$F_{dA_1-\varphi_S}(\varphi_a, \varphi_b)$	view factor from differential area dA_1 to φ-segments that extend from φ_a to φ_b as measured from dA_1
$dF_{dA_1-dA_2}$	view factor from a differential area dA_1 to differential area dA_2
F_{B-T}	view factor from an arbitrary toroidal band to its toroid
$F_{B-T}(\theta_a)$	view factor from an equatorial toroidal band that is symmetrically located about the equator from $\theta = \theta_a$ to $(2\pi - \theta_a)$ to its toroid
$F_{B-T}(\theta_a, \theta_b)$	view factor from a toroidal band that is located on a toroid from $\theta = \theta_a$ to $\theta = \theta_b$ to its toroid
F_{T-T}	self view factor of a toroid
H_c	location of sphere center on z-axis

$h_\mu, h_\theta, h_\varphi$	scale factor for toroidal coordinates μ, θ, φ
$\hat{i}, \hat{j}, \hat{k}$	Cartesian coordinate unit vectors
L	line tangent to dA_1
$\hat{n}$	normal unit vector
R	radius of circle traced out by the center of a torus ring (largest radius associated with a toroid)
R_S	radius of a sphere
r	radius of torus ring (smallest radius associated with a toroid)
$\vec{S}$ or $\underline{S}$	vector distance from dA_1 to dA_2
x, y, z	Cartesian coordinates
α	angle that r makes with x, y-plane
β	angle between normal to an area and the vector $\underline{S}$
θ_1^*	maximum θ such that $\underline{S}$ could be tangent at dA_1 and still intersect or touch the toroid
$\theta_{2min}, \theta_{2max}$	the minimum and maximum values of θ that are within the line of sight from dA_1 for a given φ
μ, θ, φ	toroidal coordinates
$\xi_\mu, \xi_\theta, \xi_\varphi$	defined by equation (10a)
ρ	ratio of toroid radii, r/R
Φ	relative φ coordinate of point 2 to point 1 (i.e., $\Phi = \varphi_2 - \varphi_1$)

Subscripts:

B_i	i^{th} toroidal band
B_{ij}	toroidal band contained by the i^{th} and j^{th} equatorial toroidal bands
1	refers to the emitting point
2	refers to the receiving point

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TABLE I. - TOROIDAL COORDINATE θ AS FUNCTION OF ANGULAR VARIABLE α

α, deg	ρ										
	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
0.	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
5.00	174.95	174.47	173.88	173.19	172.37	171.35	170.02	168.13	165.08	158.45	116.74
10.00	169.90	168.95	167.77	165.40	164.78	162.77	160.15	156.47	150.59	138.25	78.03
15.00	164.85	163.44	161.68	159.66	157.26	154.31	150.50	145.20	136.90	120.30	56.50
20.00	159.80	157.94	155.63	152.98	149.85	146.03	141.15	134.46	124.24	104.91	43.80
25.00	154.76	152.45	149.62	145.38	142.58	137.99	132.18	124.36	112.75	91.96	35.45
30.00	149.71	147.00	143.66	139.88	135.48	130.21	123.63	114.94	102.41	81.14	29.64
35.00	144.67	141.57	137.77	133.50	128.57	122.72	115.53	106.22	93.19	72.08	25.34
40.00	139.63	136.16	131.95	127.24	121.35	115.54	107.90	98.19	84.97	64.45	22.04
45.00	134.59	130.79	126.20	121.11	115.36	108.69	100.72	90.81	77.65	57.96	19.42
50.00	129.56	125.45	120.54	115.13	109.08	102.15	93.99	84.03	71.12	52.39	17.29
55.00	124.53	120.15	114.95	109.30	103.02	95.92	87.69	77.81	65.27	47.57	15.51
60.00	119.50	114.90	109.47	103.61	97.18	90.00	81.79	72.08	60.00	43.34	14.00
65.00	114.48	109.69	104.07	98.07	91.55	84.37	76.25	66.80	55.24	39.61	12.70
70.00	109.46	104.51	98.77	92.68	86.15	79.01	71.06	61.92	50.91	36.28	11.56
75.00	104.45	99.38	93.55	87.44	80.94	73.92	66.18	57.40	46.96	33.29	10.55
80.00	99.44	94.30	88.44	82.34	75.92	69.06	61.58	53.19	43.33	30.58	9.55
85.00	94.43	89.26	83.41	77.38	71.09	64.43	57.24	49.26	39.98	28.11	8.85
90.00	89.43	84.26	78.45	72.54	66.42	60.00	53.13	45.57	36.87	25.84	8.11
95.00	84.43	79.31	73.61	67.83	61.92	55.76	49.23	42.11	33.97	23.74	7.43
100.00	79.44	74.40	68.83	63.24	57.56	51.70	45.52	38.83	31.25	21.79	6.81
105.00	74.45	69.53	64.14	58.75	53.34	47.79	41.98	35.73	28.69	19.97	6.23
110.00	69.46	64.70	59.51	54.39	49.25	44.02	38.59	32.78	26.28	18.25	5.68
115.00	64.48	59.91	54.96	50.11	45.28	40.39	35.34	29.97	23.98	16.63	5.17
120.00	59.51	55.15	50.48	45.92	41.41	36.87	32.20	27.27	21.79	15.09	4.69
125.00	54.53	50.43	46.05	41.81	37.64	33.46	29.18	24.67	19.69	13.62	4.23
130.00	49.56	45.74	41.69	37.78	33.95	30.14	26.25	22.17	17.67	12.21	3.79
135.00	44.60	41.08	37.37	33.81	30.34	26.90	23.40	19.74	15.72	10.86	3.36
140.00	39.63	36.45	33.10	29.91	25.80	23.73	20.63	17.39	13.84	9.55	2.95
145.00	34.67	31.84	28.87	26.05	23.33	20.63	17.92	15.09	12.00	8.27	2.55
150.00	29.71	27.25	24.58	22.25	19.90	17.59	15.26	12.84	10.21	7.04	2.18
155.00	24.76	22.68	20.52	18.48	16.52	14.59	12.65	10.64	8.45	5.82	1.80
160.00	19.80	18.12	16.39	14.74	13.17	11.63	10.08	8.47	6.73	4.63	1.43
165.00	14.85	13.58	12.27	11.04	9.85	8.69	7.53	6.33	5.03	3.46	1.07
170.00	9.90	9.05	8.17	7.35	6.55	5.78	5.01	4.21	3.34	2.30	0.71
175.00	4.95	4.52	4.08	3.57	3.27	2.89	2.50	2.10	1.67	1.15	0.35
180.00	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.

TABLE II. - VIEW FACTOR $F_{dA_1-\varphi_S}(\varphi_a)$ FROM ELEMENT dA_1 TO SYMMETRICAL φ-SEGMENTS

α, deg	φ_a, deg	ρ										
		0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.90	0.90	0.99
0	170	0.0005589	0.0059072	0.0125797	0.0201139	0.0286042	0.0381247	0.0486894	0.0601621	0.0720384	0.0828363	0.0875703
0	160	0.0011178	0.0118106	0.0251418	0.0401826	0.0571164	0.0760855	0.0971120	0.1199200	0.1435060	0.1649520	0.1744440
0	150	0.0016767	0.0177061	0.0376678	0.0601586	0.0854412	0.1137140	0.1449970	0.1788650	0.2138370	0.2456360	0.2599310
0	140	0.0022354	0.0235890	0.0501373	0.0799906	0.1134770	0.1508340	0.1920630	0.2365880	0.2824680	0.3241990	0.3425930
0	130	0.0027940	0.0294543	0.0625268	0.0996207	0.1411100	0.1872560	0.2380200	0.2926710	0.3488500	0.3999800	0.4233060
0	120	0.0033523	0.0352952	0.0748085	0.1189820	0.1682160	0.2227710	0.2825580	0.3466940	0.4124490	0.4723620	0.5006800
0	110	0.0039103	0.0411037	0.0869479	0.1379930	0.1946460	0.2571490	0.3253530	0.3982310	0.4727520	0.5407700	0.5741230
0	100	0.0044680	0.0468690	0.0989012	0.1565570	0.2202310	0.2901360	0.3660600	0.4468590	0.5292770	0.6046890	0.6430830
0	90	0.0050251	0.0525762	0.1106120	0.1745520	0.2447700	0.3214460	0.4043180	0.4921630	0.5815810	0.6636620	0.7070460
0	80	0.0055814	0.0582050	0.1220040	0.1918260	0.2680250	0.3507640	0.4397560	0.5337430	0.6292660	0.7172920	0.7655460
0	70	0.0061366	0.0637252	0.1329760	0.2081860	0.2897210	0.3777500	0.4720010	0.5712290	0.6719890	0.7652500	0.8181600
0	60	0.0066901	0.0690921	0.1433880	0.2233950	0.3095390	0.4020450	0.5006940	0.6042940	0.7094630	0.8072620	0.8645170
0	50	0.0072410	0.0742360	0.1530500	0.2371600	0.3271300	0.4232890	0.5255030	0.6326580	0.7414640	0.8431150	0.9042910
0	40	0.0077873	0.0790459	0.1617060	0.2491370	0.3421280	0.4411440	0.5461440	0.6561030	0.7678220	0.8726430	0.9372080
0	30	0.0083247	0.0833423	0.1690260	0.2589550	0.3541850	0.4553160	0.5623950	0.6744670	0.7884190	0.8957210	0.9530430
0	20	0.0088418	0.0868462	0.1746320	0.2662540	0.3630050	0.4655830	0.5740970	0.6876450	0.8031750	0.9122630	0.9816220
0	10	0.0092975	0.0891836	0.1781600	0.2707490	0.3683760	0.4717950	0.5811520	0.6955730	0.8120450	0.9222100	0.9928190
0	-0	0.0095204	0.0900081	0.1793630	0.2722650	0.3701780	0.4738740	0.5835090	0.6982190	0.8150040	0.9255300	0.9965600
10	170	0.0005503	0.0057979	0.0122967	0.0195649	0.0276598	0.0366090	0.0463785	0.0568102	0.0675086	0.0776283	0.0850874
10	160	0.0011005	0.0115919	0.0245756	0.0390844	0.0552282	0.0730579	0.0925010	0.1132410	0.1344950	0.1546010	0.1694720
10	150	0.0016507	0.0173779	0.0368179	0.0585110	0.0826109	0.1091820	0.1381070	0.1689100	0.2004400	0.2302780	0.2524610
10	140	0.0022007	0.0231511	0.0490051	0.0777932	0.1097070	0.1448100	0.1829300	0.2234340	0.2648290	0.3040310	0.3333780
10	130	0.0027506	0.0289062	0.0611094	0.0968731	0.1364050	0.1797560	0.2266900	0.2764210	0.3271570	0.3752530	0.4115710
10	120	0.0033003	0.0346367	0.0731048	0.1156840	0.1625810	0.2138200	0.2690930	0.3274790	0.3869300	0.4433700	0.4864230
10	110	0.0038497	0.0403341	0.0849565	0.1341450	0.1880900	0.2467780	0.3098320	0.3762100	0.4436760	0.5078460	0.5573550
10	100	0.0043986	0.0459876	0.0966204	0.1521590	0.2127660	0.2783860	0.3485800	0.4222190	0.4969430	0.5681900	0.6238310
10	90	0.0049470	0.0515821	0.1080400	0.1696060	0.2364140	0.3083700	0.3849980	0.4651180	0.5463140	0.6239590	0.6853670
10	80	0.0054946	0.0570968	0.1191390	0.1863370	0.2588040	0.3364330	0.4187370	0.5045330	0.5914070	0.6747620	0.7415250
10	70	0.0060410	0.0625015	0.1298160	0.2021630	0.2796700	0.3622520	0.4494480	0.5401130	0.6318860	0.7202650	0.7919250
10	60	0.0065858	0.0677509	0.1399340	0.2168540	0.2987110	0.3854900	0.4767920	0.5715430	0.6674640	0.7601870	0.8362340
10	50	0.0071278	0.0727750	0.1493060	0.2301270	0.3155970	0.4058090	0.5004570	0.5985500	0.6979040	0.7942980	0.8741730
10	40	0.0076652	0.0774631	0.1576810	0.2416590	0.3299850	0.4228930	0.5201710	0.6209110	0.7230210	0.8224200	0.9055130
10	30	0.0081935	0.0816379	0.1647470	0.2510980	0.3415490	0.4364630	0.5357120	0.6384550	0.7426770	0.8444180	0.9300710
10	20	0.0087011	0.0850290	0.1701440	0.2581110	0.3500110	0.4463040	0.5469180	0.6510620	0.7567770	0.8601930	0.9477090
10	10	0.0091464	0.0872821	0.1735370	0.2624290	0.3551680	0.4522640	0.5536810	0.6586540	0.7652590	0.8696830	0.9583290
10	-0	0.0093623	0.0880755	0.1746950	0.2638860	0.3568990	0.4542600	0.5559420	0.6611890	0.7680900	0.8728500	0.9618760
20	170	0.0005246	0.0054765	0.0114745	0.0179917	0.0249987	0.0324201	0.0401318	0.0479553	0.0557211	0.0635400	0.0720115
20	160	0.0010491	0.0109489	0.0229306	0.0359376	0.0499077	0.0646905	0.0800356	0.0955942	0.1110370	0.1265970	0.1434560
20	150	0.0015736	0.0164129	0.0343491	0.0537901	0.0746357	0.0966577	0.1194820	0.1425990	0.1655430	0.1886910	0.2137700
20	140	0.0020979	0.0218635	0.0457087	0.0714977	0.0990850	0.1281620	0.1582350	0.1886510	0.2188390	0.2493500	0.2824000
20	130	0.0026221	0.0272951	0.0569850	0.0890026	0.1231490	0.1590320	0.1960490	0.2334270	0.2705290	0.3081100	0.3488070
20	120	0.0031460	0.0327009	0.0681493	0.1062380	0.1467070	0.1890840	0.2326710	0.2766040	0.3202270	0.3645240	0.4124710
20	110	0.0036696	0.0380721	0.0791664	0.1231250	0.1696250	0.2181190	0.2678350	0.3178580	0.3675550	0.4181580	0.4728960
20	100	0.0041928	0.0433973	0.0899917	0.1395690	0.1917460	0.2459170	0.3012660	0.3568660	0.4121490	0.4686010	0.5296140
20	90	0.0047154	0.0486609	0.1005680	0.1554540	0.2128900	0.2722410	0.3326810	0.3933120	0.4536610	0.5154680	0.5821930
20	80	0.0052371	0.0538414	0.1108200	0.1706360	0.2328500	0.2968330	0.3617900	0.4268870	0.4917630	0.5583960	0.6302410
20	70	0.0057576	0.0589077	0.1206470	0.1849420	0.2513920	0.3194230	0.3883080	0.4572970	0.5261500	0.5970540	0.6734060
20	60	0.0062763	0.0638134	0.1299170	0.1981600	0.2682560	0.3397330	0.4119560	0.4842690	0.5565430	0.6311450	0.7113850
20	50	0.0067922	0.0684880	0.1384510	0.2100420	0.2831670	0.3574880	0.4324720	0.5075510	0.5826950	0.6604120	0.7439210
20	40	0.0073030	0.0728215	0.1460250	0.2203110	0.2958450	0.3724290	0.4496180	0.5269220	0.6043880	0.6846370	0.7708040
20	30	0.0078044	0.0766444	0.1523620	0.2286810	0.3060280	0.3843200	0.4631870	0.5421930	0.6214440	0.7036490	0.7918710
20	20	0.0082839	0.0797107	0.1571690	0.2348870	0.3134860	0.3929680	0.4730090	0.5532110	0.6337240	0.7173160	0.8070000
20	10	0.0086991	0.0817237	0.1601810	0.2387080	0.3180390	0.3982200	0.4789540	0.5598660	0.6411300	0.7255510	0.8161090
20	-0	0.0088943	0.0824289	0.1612080	0.2399990	0.3195700	0.3999810	0.4809450	0.5620920	0.6436050	0.7283020	0.8191500

30	170	0.0004827	0.0049618	0.0101869	0.0155946	0.0210685	0.0264586	0.0315991	0.0363638	0.0407997	0.0454854	0.0513602
30	160	0.0009653	0.0099192	0.0203548	0.0311435	0.0420522	0.0527825	0.0630086	0.0724872	0.0813225	0.0905753	0.1023760
30	150	0.0014479	0.0148676	0.0304837	0.0465992	0.0628636	0.0788346	0.0940375	0.1081300	0.1212930	0.1352740	0.1527020
30	140	0.0019304	0.0198018	0.0405517	0.0619104	0.0834103	0.1044730	0.1244910	0.1430510	0.1604390	0.1789870	0.2019930
30	130	0.0024126	0.0247158	0.0505339	0.0770203	0.1035920	0.1295470	0.1541710	0.1770120	0.1984900	0.2215190	0.2499030
30	120	0.0028946	0.0296024	0.0604008	0.0918635	0.1232990	0.1538990	0.1828740	0.2097750	0.2351820	0.2625720	0.2960840
30	110	0.0033762	0.0344522	0.0701164	0.1063640	0.1424060	0.1773580	0.2103940	0.2411060	0.2702580	0.3018510	0.3401900
30	100	0.0038574	0.0392532	0.0796357	0.1204300	0.1607760	0.1997450	0.2365200	0.2707730	0.3034680	0.3390570	0.3818790
30	90	0.0043379	0.0439892	0.0889014	0.1339530	0.1782510	0.2208690	0.2610430	0.2985530	0.3345670	0.3738930	0.4208120
30	80	0.0048175	0.0486376	0.0978394	0.1468030	0.1946570	0.2405320	0.2837530	0.3242260	0.3633110	0.4060640	0.4566650
30	70	0.0052958	0.0531662	0.1063530	0.1588250	0.2098050	0.2585340	0.3044490	0.3475850	0.3894530	0.4352820	0.4891220
30	60	0.0057721	0.0575275	0.1143170	0.1698400	0.2234980	0.2746780	0.3229380	0.3684220	0.4127480	0.4612660	0.5178930
30	50	0.0062453	0.0616506	0.1215720	0.1796490	0.2355360	0.2887750	0.3390320	0.3865300	0.4329580	0.4837550	0.5427130
30	40	0.0067132	0.0654284	0.1279260	0.1880490	0.2457310	0.3006500	0.3525480	0.4017030	0.4498560	0.5025110	0.5633480
30	30	0.0071707	0.0687045	0.1331680	0.1948450	0.2539080	0.3101300	0.3633050	0.4137480	0.4632370	0.5173270	0.5796040
30	20	0.0076050	0.0712744	0.1370980	0.1998650	0.2599060	0.3170530	0.3711340	0.4224920	0.4729290	0.5280340	0.5913260
30	10	0.0079723	0.0729276	0.1395460	0.2029570	0.2635780	0.3212740	0.3758950	0.4277970	0.4788000	0.5345100	0.5984040
30	-0	0.0081358	0.0735021	0.1403800	0.2040040	0.2648150	0.3226930	0.3774930	0.4295760	0.4807670	0.5366770	0.6007700
40	170	0.0004260	0.0042826	0.0085430	0.0126479	0.0164447	0.0197821	0.0225490	0.0247428	0.0266188	0.0290205	0.0319388
40	160	0.0008520	0.0085606	0.0170666	0.0252510	0.0328108	0.0394479	0.0449482	0.0493153	0.0530726	0.0578784	0.0637000
40	150	0.0012778	0.0128289	0.0255502	0.0377629	0.0490177	0.0588799	0.0670476	0.0735482	0.0791978	0.0864102	0.0951041
40	140	0.0017035	0.0170823	0.0339717	0.0501339	0.0649809	0.0779569	0.0886960	0.0972746	0.1048330	0.1144500	0.1259690
40	130	0.0021290	0.0213143	0.0423055	0.0623090	0.0806097	0.0965539	0.1097420	0.1203310	0.1298210	0.1418310	0.1561060
40	120	0.0025543	0.0255171	0.0505220	0.0742257	0.0958055	0.1145410	0.1300370	0.1425620	0.1540050	0.1683780	0.1853230
40	110	0.0029791	0.0296812	0.0585851	0.0858122	0.1104610	0.1317830	0.1494320	0.1638250	0.1772270	0.1939150	0.2134190
40	100	0.0034035	0.0337938	0.0664500	0.0969843	0.1244570	0.1481430	0.1677830	0.1839890	0.1993290	0.2182580	0.2401850
40	90	0.0038271	0.0378380	0.0740607	0.1076440	0.1376680	0.1634830	0.1849580	0.2029240	0.2201470	0.2412130	0.2654040
40	80	0.0042497	0.0417904	0.0813457	0.1176770	0.1499580	0.1776690	0.2008410	0.2204950	0.2395160	0.2625830	0.2888510
40	70	0.0046709	0.0456179	0.0882147	0.1269560	0.1611940	0.1905810	0.2153240	0.2365640	0.2572590	0.2821590	0.3102970
40	60	0.0050900	0.0492727	0.0945550	0.1353420	0.1712460	0.2021270	0.2282930	0.2509830	0.2731940	0.2997320	0.3295100
40	50	0.0055057	0.0526847	0.1002330	0.1427000	0.1800100	0.2121910	0.2396260	0.2635960	0.2871340	0.3150870	0.3462600
40	40	0.0059156	0.0557529	0.1051030	0.1489130	0.1873960	0.2206790	0.2491900	0.2742390	0.2988890	0.3280140	0.3603280
40	30	0.0063144	0.0583420	0.1090360	0.1538950	0.1933120	0.2274750	0.2568430	0.2827490	0.3082750	0.3383160	0.3715120
40	20	0.0066882	0.0603042	0.1119410	0.1575580	0.1976550	0.2324570	0.2624450	0.2889690	0.3151230	0.3458180	0.3796400
40	10	0.0069929	0.0615329	0.1137360	0.1598130	0.2003200	0.2355070	0.2658690	0.2927640	0.3192950	0.3503810	0.3845750
40	-0	0.0071178	0.0619553	0.1143470	0.1605760	0.2012200	0.2365350	0.2670220	0.2940410	0.3206970	0.3519130	0.3862300
50	170	0.0003563	0.0034748	0.0066666	0.0094375	0.0116640	0.0132664	0.0142480	0.0148408	0.0156039	0.0165980	0.0177255
50	160	0.0007126	0.0069449	0.0133139	0.0188327	0.0232582	0.0264372	0.0283840	0.0295806	0.0311176	0.0331097	0.0353636
50	150	0.0010688	0.0104049	0.0199213	0.0281416	0.0347111	0.0394160	0.0422965	0.0441207	0.0464516	0.0494484	0.0528261
50	140	0.0014249	0.0138494	0.0264669	0.0373175	0.0459486	0.0521054	0.0558765	0.0583654	0.0615154	0.0655255	0.0700224
50	130	0.0017807	0.0172716	0.0329255	0.0463093	0.0568922	0.0644064	0.0690206	0.0722234	0.0762201	0.0812521	0.0868591
50	120	0.0021362	0.0206637	0.0392679	0.0550596	0.0674585	0.0762196	0.0816429	0.0856075	0.0904765	0.0965351	0.1032380
50	110	0.0024913	0.0240156	0.0454590	0.0635033	0.0775589	0.0874465	0.0936684	0.0984336	0.1041950	0.1112780	0.1190570
50	100	0.0028459	0.0273144	0.0514559	0.0715664	0.0871004	0.0979957	0.1050270	0.1106200	0.1172830	0.1253810	0.1342050
50	90	0.0031996	0.0305427	0.0572055	0.0791653	0.0959886	0.1078010	0.1156530	0.1220850	0.1296470	0.1387380	0.1485660
50	80	0.0035524	0.0336770	0.0626420	0.0862078	0.1041350	0.1168110	0.1254820	0.1327470	0.1411890	0.1512350	0.1620130
50	70	0.0039036	0.0366840	0.0676850	0.0925983	0.1114780	0.1249790	0.1344480	0.1425210	0.1518060	0.1627530	0.1744130
50	60	0.0042525	0.0395170	0.0722405	0.0982512	0.1179800	0.1322580	0.1424830	0.1513170	0.1613870	0.1731620	0.1856210
50	50	0.0045978	0.0421096	0.0762081	0.1031180	0.1236070	0.1385950	0.1495120	0.1590390	0.1698160	0.1823270	0.1954860
50	40	0.0049369	0.0443718	0.0795099	0.1071710	0.1283240	0.1439350	0.1554570	0.1655840	0.1769700	0.1901070	0.2038550
50	30	0.0052642	0.0461993	0.0821134	0.1103870	0.1320870	0.1482110	0.1602290	0.1708450	0.1827220	0.1963600	0.2105740
50	20	0.0055653	0.0475230	0.0840032	0.1127350	0.1348450	0.1513510	0.1637360	0.1747120	0.1869470	0.2009490	0.2154990
50	10	0.0057968	0.0483267	0.0851592	0.1141760	0.1365380	0.1532790	0.1658880	0.1770830	0.1895360	0.2037580	0.2185100
50	-0	0.0058812	0.0485991	0.0855510	0.1146640	0.1371110	0.1539310	0.1666150	0.1778830	0.1904090	0.2047040	0.2195240

TABLE II. - Concluded. VIEW FACTOR $F_{dA_1-\varphi_S}(\varphi_a)$ FROM ELEMENT dA_1 TO SYMMETRICAL φ-SEGMENTS

α, deg	φ_a, deg	ρ										
		0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.90	0.90	0.99
60	170	0.0002760	0.0025779	0.0046797	0.0062123	0.0071296	0.0074572	0.0075733	0.0077602	0.0080072	0.0083139	0.0086449
60	160	0.0005519	0.0051510	0.0093408	0.0123862	0.0142005	0.0148420	0.0150869	0.0154679	0.0159666	0.0165832	0.0172469
60	150	0.0008277	0.0077141	0.0139639	0.0184825	0.0211530	0.0220863	0.0224826	0.0230712	0.0238306	0.0247630	0.0257624
60	140	0.0011033	0.0102615	0.0185279	0.0244593	0.0279258	0.0291278	0.0297045	0.0305191	0.0315517	0.0328080	0.0341474
60	130	0.0013788	0.0127866	0.0230090	0.0302713	0.0344555	0.0359114	0.0367002	0.0377623	0.0390828	0.0406726	0.0423567
60	120	0.0016539	0.0152815	0.0273798	0.0358689	0.0406765	0.0423880	0.0434206	0.0447526	0.0463767	0.0483101	0.0503433
60	110	0.0019286	0.0177362	0.0316078	0.0411969	0.0465256	0.0485149	0.0498205	0.0514440	0.0533866	0.0556726	0.0580584
60	100	0.0022027	0.0201380	0.0356538	0.0461948	0.0519609	0.0542544	0.0558579	0.0577915	0.0600654	0.0627110	0.0654506
60	90	0.0024760	0.0224698	0.0394703	0.0507977	0.0569550	0.0595733	0.0614937	0.0637514	0.0663655	0.0693739	0.0724651
60	80	0.0027482	0.0247086	0.0430007	0.0549502	0.0614880	0.0644423	0.0666912	0.0692809	0.0722383	0.0756075	0.0790436
60	70	0.0030188	0.0268230	0.0461796	0.0586289	0.0655393	0.0688349	0.0714153	0.0743370	0.0776337	0.0813551	0.0851234
60	60	0.0032870	0.0287691	0.0489431	0.0618250	0.0690960	0.0727266	0.0756318	0.0788764	0.0824996	0.0865559	0.0906364
60	50	0.0035515	0.0304882	0.0512637	0.0645352	0.0721460	0.0760942	0.0793062	0.0828537	0.0867805	0.0911449	0.0955098
60	40	0.0038096	0.0319094	0.0531420	0.0667597	0.0746781	0.0789139	0.0824027	0.0862213	0.0904175	0.0950529	0.0996654
60	30	0.0040555	0.0329906	0.0545884	0.0684996	0.0766802	0.0811603	0.0848826	0.0889282	0.0933484	0.0982071	0.1030220
60	20	0.0042747	0.0337389	0.0556158	0.0697545	0.0781370	0.0828041	0.0867038	0.0909208	0.0955088	0.1005340	0.1054980
60	10	0.0044274	0.0341772	0.0562340	0.0705177	0.0790279	0.0838123	0.0878227	0.0921461	0.0968378	0.1019650	0.1070210
60	-0	0.0044750	0.0343227	0.0564419	0.0707754	0.0793291	0.0841534	0.0882012	0.0925606	0.0972873	0.1024490	0.1075290
70	170	0.0001874	0.0016315	0.0026891	0.0031571	0.0031753	0.0031782	0.0031995	0.0032346	0.0032814	0.0033392	0.0034003
70	160	0.0003748	0.0032584	0.0053613	0.0062818	0.0063170	0.0063270	0.0063726	0.0064451	0.0065406	0.0066576	0.0067808
70	150	0.0005621	0.0048757	0.0079991	0.0093411	0.0093927	0.0094180	0.0094935	0.0096076	0.0097553	0.0099342	0.0101216
70	140	0.0007492	0.0064779	0.0105834	0.0123000	0.0123721	0.0124236	0.0125369	0.0126989	0.0129035	0.0131484	0.0134027
70	130	0.0009361	0.0080588	0.0130933	0.0151224	0.0152276	0.0153181	0.0154788	0.0156959	0.0159636	0.0162792	0.0166042
70	120	0.0011227	0.0096108	0.0155044	0.0177783	0.0179348	0.0180778	0.0182962	0.0185769	0.0189142	0.0193061	0.0197057
70	110	0.0013089	0.0111245	0.0177885	0.0202476	0.0204723	0.0206812	0.0209678	0.0213205	0.0217345	0.0222084	0.0226868
70	100	0.0014944	0.0125879	0.0199125	0.0225145	0.0228229	0.0231093	0.0234741	0.0239070	0.0244044	0.0249656	0.0255267
70	90	0.0016792	0.0139852	0.0218386	0.0245669	0.0249705	0.0253455	0.0257971	0.0263174	0.0269041	0.0275574	0.0282045
70	80	0.0018629	0.0152954	0.0235404	0.0263963	0.0269041	0.0273757	0.0279210	0.0285343	0.0292147	0.0299636	0.0306989
70	70	0.0020450	0.0164904	0.0250122	0.0279975	0.0286150	0.0291883	0.0298316	0.0305414	0.0313181	0.0321640	0.0329898
70	60	0.0022247	0.0175326	0.0262550	0.0293685	0.0300975	0.0307744	0.0315169	0.0323238	0.0331966	0.0341385	0.0350514
70	50	0.0024006	0.0183855	0.0272737	0.0305109	0.0313489	0.0321272	0.0329667	0.0338678	0.0348331	0.0358669	0.0368624
70	40	0.0025702	0.0190444	0.0280774	0.0314293	0.0323694	0.0332425	0.0341722	0.0351604	0.0362108	0.0373286	0.0383989
70	30	0.0027278	0.0195218	0.0286786	0.0321312	0.0331611	0.0341173	0.0351255	0.0361891	0.0373127	0.0385021	0.0396360
70	20	0.0028595	0.0198372	0.0290925	0.0326257	0.0337270	0.0347487	0.0358185	0.0369408	0.0381210	0.0393653	0.0405478
70	10	0.0029358	0.0200133	0.0293345	0.0329209	0.0340686	0.0351326	0.0362417	0.0374013	0.0386173	0.0398963	0.0411092
70	-0	0.0029557	0.0200699	0.0294146	0.0330196	0.0341834	0.0352620	0.0363847	0.0375570	0.0387852	0.0400760	0.0412968
80	170	0.0000935	0.0006729	0.0007985	0.0007873	0.0007813	0.0007781	0.0007767	0.0007769	0.0007781	0.0007804	0.0007831
80	160	0.0001871	0.0013415	0.0015865	0.0015653	0.0015541	0.0015482	0.0015461	0.0015467	0.0015496	0.0015544	0.0015601
80	150	0.0002805	0.0020015	0.0023539	0.0023251	0.0023100	0.0023027	0.0023006	0.0023025	0.0023077	0.0023157	0.0023249
80	140	0.0003737	0.0026478	0.0030914	0.0030579	0.0030412	0.0030339	0.0030332	0.0030376	0.0030460	0.0030580	0.0030715
80	130	0.0004668	0.0032750	0.0037905	0.0037560	0.0037400	0.0037349	0.0037372	0.0037453	0.0037583	0.0037753	0.0037939
80	120	0.0005595	0.0038762	0.0044439	0.0044122	0.0043998	0.0043988	0.0044061	0.0044196	0.0044384	0.0044618	0.0044864
80	110	0.0006518	0.0044431	0.0050455	0.0050203	0.0050143	0.0050199	0.0050339	0.0050546	0.0050807	0.0051118	0.0051435
80	100	0.0007436	0.0049652	0.0055905	0.0055754	0.0055785	0.0055929	0.0056156	0.0056450	0.0056800	0.0057200	0.0057600
80	90	0.0008346	0.0054299	0.0060754	0.0060735	0.0060881	0.0061134	0.0061466	0.0061863	0.0062315	0.0062818	0.0063310
80	80	0.0009246	0.0058307	0.0064982	0.0065118	0.0065400	0.0065779	0.0066231	0.0066745	0.0067312	0.0067928	0.0068521
80	70	0.0010130	0.0061673	0.0068580	0.0068889	0.0069322	0.0069840	0.0070424	0.0071065	0.0071756	0.0072493	0.0073195
80	60	0.0010992	0.0064412	0.0071557	0.0072048	0.0072640	0.0073305	0.0074028	0.0074802	0.0075622	0.0076486	0.0077299
80	50	0.0011819	0.0066555	0.0073932	0.0074606	0.0075360	0.0076174	0.0077037	0.0077945	0.0078894	0.0079884	0.0080808
80	40	0.0012585	0.0068147	0.0075741	0.0076591	0.0077501	0.0078458	0.0079456	0.0080492	0.0081565	0.0082674	0.0083703
80	30	0.0013239	0.0069249	0.0077036	0.0078045	0.0079095	0.0080182	0.0081301	0.0082452	0.0083636	0.0084852	0.0085974
80	20	0.0013675	0.0069933	0.0077880	0.0079020	0.0080186	0.0081378	0.0082596	0.0083841	0.0085113	0.0086414	0.0087611
80	10	0.0013867	0.0070287	0.0078345	0.0079574	0.0080819	0.0082083	0.0083366	0.0084672	0.0086003	0.0087360	0.0088605
80	-0	0.0013909	0.0070393	0.0078492	0.0079754	0.0081028	0.0082316	0.0083623	0.0084951	0.0086302	0.0087678	0.0088931

TABLE III. - VIEW FACTORS $F^{(upper)}_{dA_1-B}(\varphi_a)$ AND $F^{(lower)}_{dA_1-B}(\varphi_b)$ FROM ELEMENT dA_1 TO EQUATORIAL TOROIDAL BANDS ABOVE AND BELOW EQUATOR

α_1	α_2	ρ											
		0.01			0.10			0.20			0.30		
		Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band
0	10	0.001735	0.000868	0.000868	0.018819	0.009409	0.009409	0.041654	0.020827	0.020827	0.070245	0.035123	0.035123
0	20	0.003404	0.001702	0.001702	0.036051	0.018025	0.018026	0.078647	0.039324	0.039324	0.130386	0.065193	0.065193
0	30	0.004944	0.002472	0.002472	0.051335	0.025668	0.025668	0.110158	0.055079	0.055079	0.179464	0.089732	0.089732
0	40	0.006307	0.003153	0.003153	0.064279	0.032139	0.032139	0.135736	0.067868	0.067868	0.217353	0.108676	0.108676
0	50	0.007458	0.003729	0.003729	0.074656	0.037328	0.037328	0.155192	0.077596	0.077596	0.244489	0.122245	0.122245
0	60	0.008375	0.004187	0.004187	0.082332	0.041166	0.041166	0.168629	0.084315	0.084315	0.261728	0.130864	0.130864
0	70	0.009030	0.004515	0.004515	0.087291	0.043645	0.043645	0.176399	0.088199	0.088199	0.270312	0.135156	0.135156
0	80	0.009410	0.004705	0.004705	0.089658	0.044829	0.044829	0.179241	0.089620	0.089620	0.272265	0.136132	0.136132
0	90	0.009521	0.004761	0.004761	0.090008	0.045004	0.045004	0.179364	0.089682	0.089682	0.272265	0.136132	0.136132
10	10	0.001701	0.000854	0.000847	0.017987	0.009249	0.008738	0.039145	0.020427	0.018718	0.064695	0.034353	0.030341
10	20	0.003338	0.001683	0.001656	0.034974	0.018218	0.016757	0.075459	0.040237	0.035222	0.123646	0.067579	0.055966
10	30	0.004852	0.002453	0.002399	0.049985	0.026156	0.023829	0.106429	0.057240	0.049189	0.171869	0.095261	0.076608
10	40	0.006196	0.003137	0.003058	0.062723	0.032921	0.029802	0.131574	0.071143	0.060430	0.209182	0.116869	0.092313
10	50	0.007332	0.003715	0.003617	0.072934	0.038362	0.034572	0.150734	0.081835	0.068899	0.235992	0.132641	0.103351
10	60	0.008233	0.004172	0.004061	0.080495	0.042417	0.038077	0.163991	0.089333	0.074657	0.253114	0.142965	0.110149
10	70	0.008879	0.004501	0.004378	0.085382	0.045067	0.040315	0.171687	0.093803	0.077884	0.261746	0.148450	0.113296
10	80	0.009254	0.004693	0.004561	0.087722	0.046372	0.041350	0.174542	0.095593	0.078948	0.263873	0.150048	0.113825
10	90	0.009363	0.004749	0.004613	0.088075	0.046591	0.041484	0.174693	0.095722	0.078971	0.263886	0.150061	0.113825
20	10	0.001603	0.000808	0.000795	0.016135	0.008288	0.007847	0.033689	0.017642	0.016047	0.053098	0.028378	0.024720
20	20	0.003149	0.001598	0.001551	0.031808	0.016799	0.015009	0.066426	0.036318	0.030108	0.104880	0.059544	0.045336
20	30	0.004586	0.002339	0.002247	0.046093	0.024783	0.021310	0.095765	0.053848	0.041917	0.150545	0.088814	0.061731
20	40	0.005869	0.003003	0.002867	0.058195	0.031579	0.026618	0.119660	0.068305	0.051354	0.186129	0.112098	0.074031
20	50	0.006956	0.003565	0.003392	0.067922	0.037085	0.030837	0.137924	0.079528	0.058396	0.211928	0.129406	0.082522
20	60	0.007816	0.004009	0.003808	0.075132	0.041209	0.033922	0.150636	0.087522	0.063114	0.228650	0.141054	0.087596
20	70	0.008432	0.004328	0.004104	0.079809	0.043939	0.035870	0.158100	0.092424	0.065677	0.237364	0.147597	0.089768
20	80	0.008790	0.004515	0.004275	0.082068	0.045324	0.036745	0.160979	0.094547	0.066432	0.239914	0.149890	0.090024
20	90	0.008895	0.004571	0.004324	0.082429	0.045586	0.036842	0.161207	0.094770	0.066437	0.239999	0.149975	0.090024
30	10	0.001449	0.000732	0.000717	0.013887	0.007153	0.006733	0.027546	0.014471	0.013075	0.040956	0.022027	0.018929
30	20	0.002853	0.001452	0.001400	0.027347	0.014435	0.012862	0.054357	0.029916	0.024441	0.081079	0.046574	0.034505
30	30	0.004166	0.002135	0.002031	0.040020	0.021777	0.018242	0.079683	0.045758	0.033925	0.119425	0.072694	0.046731
30	40	0.005345	0.002752	0.002593	0.051127	0.028368	0.022758	0.101532	0.060085	0.041446	0.152118	0.096334	0.055784
30	50	0.006347	0.003279	0.003068	0.060047	0.033709	0.026338	0.118335	0.071323	0.047012	0.176152	0.114217	0.061936
30	60	0.007141	0.003697	0.003445	0.066688	0.037745	0.028943	0.130129	0.079434	0.050694	0.192067	0.126556	0.065511
30	70	0.007708	0.003996	0.003712	0.071016	0.040445	0.030572	0.137176	0.084539	0.052637	0.200744	0.133819	0.066925
30	80	0.008039	0.004172	0.003867	0.073135	0.041852	0.031283	0.140048	0.086901	0.053147	0.203773	0.136735	0.067038
30	90	0.008136	0.004226	0.003910	0.073502	0.042150	0.031352	0.140380	0.087233	0.053147	0.204004	0.136966	0.067038

40	10	0.001253	0.000632	0.000621	0.011339	0.005849	0.005490	0.021190	0.011183	0.010007	0.029376	0.015918	0.013457
40	20	0.002472	0.001258	0.001215	0.022357	0.011885	0.010472	0.041851	0.023213	0.018638	0.058306	0.033947	0.024358
40	30	0.003618	0.001855	0.001763	0.032710	0.017882	0.014828	0.061451	0.035672	0.025779	0.086233	0.053443	0.032790
40	40	0.004650	0.002400	0.002250	0.042116	0.023640	0.018476	0.079419	0.048015	0.031404	0.112430	0.073478	0.038951
40	50	0.005533	0.002870	0.002663	0.049970	0.028611	0.021360	0.094214	0.058677	0.035536	0.133885	0.090803	0.043082
40	60	0.006236	0.003247	0.002989	0.055825	0.032375	0.023450	0.104722	0.066479	0.038243	0.148451	0.103020	0.045431
40	70	0.006738	0.003517	0.003221	0.059674	0.034925	0.024749	0.111133	0.071496	0.039636	0.156790	0.110494	0.045296
40	80	0.007031	0.003677	0.003354	0.061590	0.036289	0.025302	0.113915	0.073951	0.039964	0.160157	0.113816	0.046341
40	90	0.007118	0.003726	0.003392	0.061955	0.036607	0.025349	0.114347	0.074383	0.039964	0.160576	0.114235	0.046341
50	10	0.001030	0.000518	0.000511	0.008656	0.004477	0.004179	0.015010	0.007967	0.007044	0.019071	0.010445	0.008626
50	20	0.002029	0.001030	0.000999	0.017064	0.009113	0.007951	0.029695	0.016647	0.013048	0.037999	0.022544	0.015455
50	30	0.002969	0.001520	0.001449	0.024997	0.013759	0.011238	0.043697	0.025735	0.017962	0.056534	0.035908	0.020626
50	40	0.003820	0.001971	0.001849	0.032202	0.018222	0.013980	0.056646	0.034847	0.021799	0.074242	0.049906	0.024336
50	50	0.004552	0.002366	0.002187	0.038471	0.022328	0.016143	0.068105	0.043507	0.024598	0.090503	0.063711	0.026792
50	60	0.005139	0.002685	0.002454	0.043366	0.025657	0.017708	0.076960	0.050540	0.026420	0.103103	0.074933	0.028170
50	70	0.005561	0.002917	0.002644	0.046605	0.027928	0.018677	0.082496	0.055152	0.027344	0.110665	0.082013	0.028652
50	80	0.005808	0.003054	0.002754	0.048253	0.029171	0.019082	0.085054	0.057512	0.027543	0.114077	0.085408	0.028659
50	90	0.005881	0.003097	0.002784	0.048599	0.029486	0.019113	0.085551	0.058008	0.027543	0.114564	0.085995	0.028559
60	10	0.000781	0.000394	0.000387	0.005952	0.003090	0.002862	0.009326	0.004999	0.004327	0.010448	0.005835	0.004612
60	20	0.001538	0.000782	0.000756	0.011747	0.006323	0.005424	0.018498	0.010560	0.007938	0.020959	0.012872	0.008097
60	30	0.002248	0.001152	0.001096	0.017224	0.009584	0.007641	0.027332	0.016496	0.010837	0.031542	0.020924	0.010619
60	40	0.002891	0.001494	0.001397	0.022231	0.012750	0.009480	0.035609	0.022547	0.013062	0.041975	0.029619	0.012356
60	50	0.003448	0.001796	0.001652	0.026601	0.015675	0.010926	0.043060	0.028393	0.014667	0.051887	0.038415	0.013471
60	60	0.003898	0.002044	0.001854	0.030167	0.018216	0.011971	0.049364	0.033657	0.015707	0.060733	0.045642	0.014091
60	70	0.004225	0.002227	0.001998	0.032707	0.020087	0.012620	0.053776	0.037541	0.016234	0.067024	0.052719	0.014306
60	80	0.004418	0.002335	0.002081	0.034019	0.021130	0.012889	0.055943	0.039601	0.016342	0.070111	0.055799	0.014311
60	90	0.004475	0.002371	0.002104	0.034323	0.021415	0.012908	0.056442	0.040100	0.016342	0.070775	0.056464	0.014311
70	10	0.000513	0.000259	0.000254	0.003365	0.001762	0.001603	0.004407	0.002419	0.001988	0.003789	0.002239	0.001549
70	20	0.001011	0.000516	0.000496	0.006652	0.003641	0.003012	0.008798	0.005240	0.003558	0.007788	0.005245	0.002543
70	30	0.001479	0.000751	0.000717	0.009780	0.005569	0.004211	0.013131	0.008381	0.004750	0.012123	0.008988	0.003135
70	40	0.001901	0.000987	0.000914	0.012555	0.007473	0.005192	0.017321	0.011703	0.005618	0.016774	0.013313	0.003461
70	50	0.002267	0.001187	0.001080	0.015214	0.009258	0.005955	0.021229	0.015011	0.006218	0.021553	0.017925	0.003628
70	60	0.002566	0.001353	0.001213	0.017334	0.010828	0.006505	0.024663	0.018067	0.006597	0.025120	0.022415	0.003705
70	70	0.002785	0.001478	0.001307	0.018927	0.012078	0.006849	0.027397	0.020606	0.006790	0.030031	0.025300	0.003731
70.	80	0.002916	0.001554	0.001362	0.019833	0.012844	0.006994	0.028995	0.022164	0.006831	0.032418	0.028586	0.003732
70	90	0.002956	0.001579	0.001377	0.020070	0.013067	0.007003	0.029415	0.022584	0.006831	0.033020	0.029288	0.003732
80	10	0.000239	0.000121	0.000118	0.001075	0.000582	0.000493	0.000701	0.000442	0.000259	0.000052	0.000052	0.
80	20	0.000472	0.000242	0.000230	0.002139	0.001243	0.000896	0.001481	0.001099	0.000383	0.000350	0.000350	0.
80	30	0.000691	0.000359	0.000332	0.003178	0.001964	0.001214	0.002398	0.001975	0.000423	0.001014	0.001014	0.
80	40	0.000890	0.000457	0.000422	0.004163	0.002713	0.001456	0.003464	0.003037	0.000428	0.002075	0.002075	0.
80	50	0.001062	0.000554	0.000498	0.005081	0.003449	0.001632	0.004638	0.004211	0.000428	0.003463	0.003463	0.
80	60	0.001203	0.000644	0.000558	0.005874	0.004122	0.001752	0.005814	0.005386	0.000428	0.005013	0.005013	0.
80	70	0.001307	0.000706	0.000601	0.006501	0.004676	0.001826	0.006852	0.006424	0.000428	0.006485	0.006485	0.
80	80	0.001371	0.000744	0.000627	0.006909	0.005051	0.001858	0.007596	0.007169	0.000428	0.007596	0.007596	0.
80	90	0.001391	0.000757	0.000634	0.007037	0.005178	0.001861	0.007849	0.007422	0.000428	0.007975	0.007975	0.

TABLE III. - Continued. VIEW FACTORS $F_{dA_1-B}^{(upper)}(\varphi_a)$ AND $F_{dA_1-B}^{(lower)}(\varphi_b)$ FROM ELEMENT dA_1 TO EQUATORIAL TOROIDAL BANDS ABOVE AND BELOW EQUATOR

α_1	α_2	ρ											
		0.40			0.50			0.60			0.70		
		Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band
0	10	0.107156	0.053578	0.053578	0.156595	0.078297	0.078297	0.226093	0.113047	0.113047	0.330200	0.165100	0.165100
0	20	0.194720	0.097360	0.097360	0.276683	0.138342	0.138342	0.383580	0.191790	0.191790	0.525100	0.262500	0.262500
0	30	0.262405	0.131203	0.131203	0.362753	0.181376	0.181376	0.484315	0.242158	0.242158	0.628400	0.314200	0.314200
0	40	0.311447	0.155723	0.155723	0.419914	0.209957	0.209957	0.543179	0.271589	0.271589	0.677600	0.338800	0.338800
0	50	0.343936	0.171968	0.171968	0.453889	0.226945	0.226945	0.572813	0.286406	0.286406	0.695900	0.347900	0.347900
0	60	0.362354	0.181177	0.181177	0.470050	0.235025	0.235025	0.582907	0.291454	0.291454	0.698200	0.349100	0.349100
0	70	0.369557	0.184778	0.184778	0.473872	0.236936	0.236936	0.583509	0.291754	0.291754	0.698200	0.349100	0.349100
0	80	0.370178	0.185089	0.185089	0.473874	0.236937	0.236937	0.583509	0.291754	0.291754	0.698200	0.349100	0.349100
0	90	0.370178	0.185089	0.185089	0.473874	0.236937	0.236937	0.583509	0.291754	0.291754	0.698200	0.349100	0.349100
10	10	0.096211	0.052216	0.043995	0.135896	0.075930	0.059966	0.186691	0.108840	0.077851	0.251700	0.157100	0.094560
10	20	0.182046	0.102895	0.079151	0.253952	0.149697	0.104255	0.343558	0.214800	0.128758	0.455700	0.310900	0.144800
10	30	0.248706	0.142984	0.105722	0.339517	0.204647	0.134870	0.446644	0.287093	0.159551	0.571100	0.401800	0.169300
10	40	0.297281	0.172773	0.124508	0.397098	0.242672	0.154426	0.508709	0.332226	0.176483	0.629900	0.450000	0.179900
10	50	0.329712	0.193159	0.136553	0.431946	0.266538	0.165408	0.541350	0.357229	0.184120	0.654600	0.471900	0.182800
10	60	0.348350	0.205369	0.142981	0.449147	0.279151	0.169995	0.553947	0.368055	0.185892	0.661000	0.478200	0.182800
10	70	0.355941	0.210864	0.145076	0.454061	0.283441	0.170620	0.555942	0.370048	0.185894	0.661200	0.478400	0.182800
10	80	0.356899	0.211760	0.145139	0.454260	0.283640	0.170620	0.555942	0.370048	0.185894	0.661200	0.478400	0.182800
10	90	0.356899	0.211760	0.145139	0.454260	0.283640	0.170620	0.555942	0.370048	0.185894	0.661200	0.478400	0.182800
20	10	0.074341	0.040762	0.033579	0.096670	0.054913	0.041758	0.117450	0.070271	0.047180	0.129500	0.084030	0.045480
20	20	0.147497	0.087667	0.059830	0.193769	0.122235	0.071535	0.241480	0.165057	0.076423	0.285300	0.217400	0.067920
20	30	0.211011	0.131759	0.079251	0.277093	0.185633	0.091460	0.347954	0.254634	0.093321	0.422700	0.344400	0.078270
20	40	0.258021	0.165348	0.092673	0.335078	0.231329	0.103749	0.416565	0.314440	0.102125	0.502200	0.419900	0.082310
20	50	0.290068	0.189053	0.101015	0.371767	0.261490	0.110277	0.456099	0.350450	0.105649	0.542600	0.459600	0.083010
20	60	0.309148	0.203952	0.105196	0.391439	0.278824	0.112615	0.474813	0.368687	0.106127	0.559100	0.476100	0.083010
20	70	0.317696	0.211415	0.106281	0.398952	0.286190	0.112762	0.480709	0.374583	0.106127	0.562100	0.479100	0.083010
20	80	0.319568	0.213284	0.106284	0.399981	0.287219	0.112762	0.480945	0.374819	0.106127	0.562100	0.479100	0.083010
20	90	0.319569	0.213286	0.106284	0.399981	0.287219	0.112762	0.480945	0.374819	0.106127	0.562100	0.479100	0.083010
30	10	0.053265	0.029515	0.023750	0.062653	0.036171	0.026482	0.065845	0.040333	0.025512	0.057970	0.038800	0.019170
30	20	0.106200	0.064293	0.041907	0.127015	0.082291	0.044725	0.138930	0.098386	0.040544	0.135400	0.107500	0.027950
30	30	0.158077	0.103015	0.055062	0.193370	0.136809	0.056561	0.222097	0.173235	0.048862	0.241000	0.209300	0.031770
30	40	0.202070	0.138103	0.063966	0.249961	0.186325	0.063636	0.294500	0.241501	0.052999	0.336000	0.302900	0.033130
30	50	0.232923	0.163573	0.069349	0.287719	0.220506	0.067213	0.340129	0.285651	0.054478	0.391500	0.358200	0.033290
30	60	0.252145	0.180254	0.071891	0.309798	0.241497	0.068301	0.365186	0.310598	0.054587	0.419300	0.385100	0.033290
30	70	0.261721	0.189320	0.072401	0.320127	0.251804	0.068323	0.375831	0.321243	0.054587	0.428900	0.395600	0.033290
30	80	0.264728	0.192327	0.072401	0.322691	0.254368	0.068323	0.377493	0.322905	0.054587	0.429600	0.396300	0.033290
30	90	0.264815	0.192414	0.072401	0.322694	0.254371	0.068323	0.377493	0.322905	0.054587	0.429600	0.396300	0.033290

40	10	0.034977	0.019621	0.015356	0.036555	0.021506	0.015049	0.032468	0.020463	0.012004	0.021990	0.015410	0.006579
40	20	0.070174	0.043385	0.025790	0.075151	0.050135	0.024976	0.070717	0.052154	0.018562	0.055410	0.045320	0.009336
40	30	0.105418	0.070543	0.034875	0.116717	0.085548	0.031169	0.117843	0.095897	0.021945	0.107600	0.097510	0.009988
40	40	0.139922	0.099690	0.040232	0.160403	0.125654	0.034750	0.172939	0.149404	0.023534	0.178100	0.157800	0.010260
40	50	0.168329	0.124931	0.043398	0.196962	0.160470	0.036493	0.220108	0.196048	0.024060	0.239700	0.229400	0.010290
40	60	0.186857	0.142035	0.044822	0.219953	0.183001	0.036953	0.248681	0.224599	0.024083	0.274900	0.254500	0.010290
40	70	0.196976	0.151926	0.045048	0.232160	0.195206	0.036954	0.262949	0.238867	0.024083	0.290700	0.280400	0.010290
40	80	0.200903	0.155855	0.045048	0.236356	0.199401	0.036954	0.266979	0.242897	0.024083	0.294000	0.283800	0.010290
40	90	0.201220	0.156172	0.045048	0.236526	0.199572	0.036954	0.267021	0.242939	0.024083	0.294000	0.283800	0.010290
50	10	0.020303	0.011597	0.008706	0.018178	0.011030	0.007148	0.012705	0.008481	0.004223	0.005355	0.004316	0.001039
50	20	0.041120	0.026203	0.014917	0.038237	0.026756	0.011481	0.029396	0.023323	0.006073	0.015820	0.015710	0.001110
50	30	0.062626	0.043490	0.019136	0.061244	0.047283	0.013961	0.052631	0.045838	0.006793	0.039180	0.038070	0.001110
50	40	0.084456	0.062619	0.021837	0.086995	0.071704	0.015291	0.082550	0.075506	0.007044	0.073610	0.072500	0.001110
50	50	0.105681	0.082286	0.023395	0.114016	0.098108	0.015908	0.116838	0.109725	0.007113	0.117000	0.115900	0.001110
50	60	0.122375	0.098296	0.024079	0.135718	0.119657	0.016061	0.144874	0.137759	0.007115	0.152700	0.151600	0.001110
50	70	0.132171	0.108000	0.024170	0.148219	0.132157	0.016061	0.160351	0.153235	0.007115	0.171300	0.170200	0.001110
50	80	0.136512	0.112342	0.024170	0.153389	0.137328	0.016061	0.166186	0.159071	0.007115	0.177600	0.176500	0.001110
50	90	0.137111	0.112940	0.024170	0.153931	0.137869	0.016061	0.166615	0.159500	0.007115	0.177900	0.176800	0.001110
60	10	0.009326	0.005534	0.003792	0.006261	0.004119	0.002141	0.002316	0.001933	0.000383	0.000139	0.000139	0.
60	20	0.019277	0.013051	0.006225	0.014040	0.010962	0.003078	0.007286	0.006901	0.000386	0.002382	0.002382	0.
60	30	0.030212	0.022516	0.007696	0.024327	0.020923	0.003404	0.016571	0.016185	0.000386	0.009626	0.009626	0.
60	40	0.042058	0.033524	0.008533	0.037242	0.033757	0.003485	0.030506	0.030120	0.000386	0.023620	0.023620	0.
60	50	0.054269	0.045295	0.008974	0.051969	0.048473	0.003496	0.048053	0.047667	0.000386	0.043720	0.043720	0.
60	60	0.065920	0.056759	0.009161	0.066987	0.063491	0.003496	0.066987	0.066602	0.000386	0.066990	0.066990	0.
60	70	0.074426	0.065241	0.009186	0.078125	0.074629	0.003496	0.081077	0.080692	0.000386	0.084380	0.084380	0.
60	80	0.078539	0.069354	0.009186	0.083282	0.079786	0.003496	0.087289	0.086903	0.000386	0.091650	0.091650	0.
60	90	0.079331	0.070145	0.009186	0.084153	0.080657	0.003496	0.088201	0.087816	0.000386	0.092560	0.092560	0.
70	10	0.002040	0.001411	0.000629	0.000334	0.000332	0.000002	0.	0.	0.	0.	0.	0.
70	20	0.004707	0.003901	0.000806	0.001759	0.001757	0.000002	0.000273	0.000273	0.	0.	0.	0.
70	30	0.008441	0.007626	0.000816	0.004841	0.004839	0.000002	0.002179	0.002179	0.	0.000574	0.000574	0.
70	40	0.013320	0.012504	0.000816	0.009766	0.009764	0.000002	0.006592	0.006592	0.	0.003912	0.003912	0.
70	50	0.018994	0.018178	0.000816	0.016213	0.016211	0.000002	0.013459	0.013459	0.	0.010770	0.010770	0.
70	60	0.024860	0.024045	0.000816	0.023383	0.023381	0.000002	0.021832	0.021832	0.	0.020200	0.020200	0.
70	70	0.030154	0.029338	0.000816	0.030154	0.030152	0.000002	0.030154	0.030154	0.	0.030150	0.030150	0.
70	80	0.033411	0.032595	0.000816	0.034331	0.034329	0.000002	0.035305	0.035305	0.	0.036340	0.036340	0.
70	90	0.034183	0.033367	0.000816	0.035262	0.035260	0.000002	0.036385	0.036385	0.	0.037560	0.037560	0.
80	10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
80	20	0.000004	0.000004	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
80	30	0.000253	0.000253	0.	0.000006	0.000006	0.	0.	0.	0.	0.	0.	0.
80	40	0.001059	0.001059	0.	0.000400	0.000400	0.	0.000070	0.000070	0.	0.	0.	0.
80	50	0.002456	0.002456	0.	0.001617	0.001617	0.	0.000953	0.000953	0.	0.000467	0.000467	0.
80	60	0.004258	0.004258	0.	0.003549	0.003549	0.	0.002890	0.002890	0.	0.002284	0.002284	0.
80	70	0.006118	0.006118	0.	0.005751	0.005751	0.	0.005384	0.005384	0.	0.005016	0.005016	0.
80	80	0.007596	0.007596	0.	0.007596	0.007596	0.	0.007596	0.007596	0.	0.007596	0.007596	0.
80	90	0.008103	0.008103	0.	0.008232	0.008232	0.	0.008362	0.008362	0.	0.008495	0.008495	0.

TABLE III. - Concluded. VIEW FACTORS $F_{dA_1-B}^{(upper)}(\varphi_a)$ AND $F_{dA_1-B}^{(lower)}(\varphi_b)$ FROM ELEMENT dA_1 TO EQUATORIAL TOROIDAL BANDS ABOVE AND BELOW EQUATOR

α_1	α_2	ρ								
		0.80			0.90			0.99		
		Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band
0	10	0.498800	0.249400	0.249400	0.780300	0.390100	0.390100	0.997300	0.498600	0.498600
0	20	0.708000	0.354000	0.354000	0.903300	0.451700	0.451700	0.997300	0.498700	0.498700
0	30	0.786200	0.393100	0.393100	0.924400	0.462200	0.462200	0.997300	0.498700	0.498700
0	40	0.811500	0.405800	0.405800	0.925500	0.462800	0.462800	0.997300	0.498700	0.498700
0	50	0.815000	0.407500	0.407500	0.925500	0.462800	0.462800	0.997300	0.498700	0.498700
0	60	0.815000	0.407500	0.407500	0.925500	0.462800	0.462800	0.997300	0.498700	0.498700
0	70	0.815000	0.407500	0.407500	0.925500	0.462800	0.462800	0.997300	0.498700	0.498700
0	80	0.815000	0.407500	0.407500	0.925500	0.462800	0.462800	0.997300	0.498700	0.498700
0	90	0.815000	0.407500	0.407500	0.925500	0.462800	0.462800	0.997300	0.498700	0.498700
10	10	0.329700	0.232500	0.097140	0.397500	0.347600	0.049850	0.413200	0.413200	0.000006
10	20	0.595100	0.463300	0.131800	0.771300	0.714000	0.057280	0.956500	0.956500	0.000006
10	30	0.710400	0.567000	0.143500	0.854800	0.796600	0.058140	0.961900	0.961900	0.000006
10	40	0.754500	0.608200	0.146300	0.872000	0.813800	0.058140	0.961900	0.961900	0.000006
10	50	0.767100	0.620800	0.146300	0.872900	0.814700	0.058140	0.961900	0.961900	0.000006
10	60	0.768100	0.621800	0.146300	0.872900	0.814700	0.058140	0.961900	0.961900	0.000006
10	70	0.768100	0.621800	0.146300	0.872900	0.814700	0.058140	0.961900	0.961900	0.000006
10	80	0.768100	0.621800	0.146300	0.872900	0.814700	0.058140	0.961900	0.961900	0.000006
10	90	0.768100	0.621800	0.146300	0.872900	0.814700	0.058140	0.961900	0.961900	0.000006
20	10	0.116500	0.086090	0.030420	0.056800	0.050600	0.006193	0.000896	0.000896	0.
20	20	0.316500	0.276000	0.040510	0.328500	0.321400	0.007036	0.329000	0.329000	0.
20	30	0.504200	0.460500	0.043680	0.610300	0.603200	0.007098	0.766000	0.766000	0.
20	40	0.594800	0.550600	0.044180	0.701900	0.694800	0.007098	0.816500	0.816500	0.
20	50	0.632600	0.588400	0.044180	0.726300	0.719200	0.007098	0.819200	0.819200	0.
20	60	0.643200	0.599000	0.044180	0.728300	0.721200	0.007098	0.819200	0.819200	0.
20	70	0.643600	0.599400	0.044180	0.728300	0.721200	0.007098	0.819200	0.819200	0.
20	80	0.643600	0.599400	0.044180	0.728300	0.721200	0.007098	0.819200	0.819200	0.
20	90	0.643600	0.599400	0.044180	0.728300	0.721200	0.007098	0.819200	0.819200	0.
30	10	0.035440	0.027160	0.008282	0.007341	0.007044	0.000297	0.	0.	0.
30	20	0.110000	0.099370	0.010630	0.061810	0.061510	0.000297	0.013120	0.013120	0.
30	30	0.249100	0.237900	0.011260	0.250000	0.249700	0.000297	0.250000	0.250000	0.
30	40	0.378900	0.367600	0.011330	0.434700	0.434400	0.000297	0.515100	0.515100	0.
30	50	0.445800	0.434500	0.011330	0.510500	0.510200	0.000297	0.588100	0.588100	0.
30	60	0.474100	0.462800	0.011330	0.534400	0.534100	0.000297	0.600800	0.600800	0.
30	70	0.480700	0.469400	0.011330	0.536700	0.536400	0.000297	0.600800	0.600800	0.
30	80	0.480800	0.469400	0.011330	0.536700	0.536400	0.000297	0.600800	0.600800	0.
30	90	0.480800	0.469400	0.011330	0.536700	0.536400	0.000297	0.600800	0.600800	0.

40	10	0.008063	0.006864	0.001199	0.000280	0.000280	0.	0.	0.	0.
40	20	0.031860	0.030610	0.001254	0.010000	0.010000	0.	0.000416	0.000416	0.
40	30	0.087340	0.086080	0.001254	0.060630	0.060630	0.	0.031980	0.031980	0.
40	40	0.178600	0.177400	0.001254	0.178600	0.178600	0.	0.178600	0.178600	0.
40	50	0.259800	0.258600	0.001254	0.286500	0.286500	0.	0.320300	0.320300	0.
40	60	0.302000	0.300700	0.001254	0.335500	0.335500	0.	0.373800	0.373800	0.
40	70	0.318500	0.317200	0.001254	0.350900	0.350900	0.	0.386100	0.386100	0.
40	80	0.320700	0.319400	0.001254	0.351900	0.351900	0.	0.386200	0.386200	0.
40	90	0.320700	0.319400	0.001254	0.351900	0.351900	0.	0.386200	0.386200	0.
50	10	0.000492	0.000492	0.	0.	0.	0.	0.	0.	0.
50	20	0.006124	0.006124	0.	0.000597	0.000597	0.	0.	0.	0.
50	30	0.025000	0.025000	0.	0.011920	0.011920	0.	0.003283	0.003283	0.
50	40	0.062830	0.062830	0.	0.050100	0.050100	0.	0.036840	0.036840	0.
50	50	0.117000	0.117000	0.	0.117000	0.117000	0.	0.117000	0.117000	0.
50	60	0.162000	0.162000	0.	0.173500	0.173500	0.	0.186400	0.186400	0.
50	70	0.183900	0.183900	0.	0.198600	0.198600	0.	0.214200	0.214200	0.
50	80	0.190300	0.190300	0.	0.204700	0.204700	0.	0.219500	0.219500	0.
50	90	0.190400	0.190400	0.	0.204700	0.204700	0.	0.219500	0.219500	0.
60	10	0.	0.	0.	0.	0.	0.	0.	0.	0.
60	20	0.000186	0.000186	0.	0.	0.	0.	0.	0.	0.
60	30	0.004172	0.004172	0.	0.000873	0.000873	0.	0.000003	0.000003	0.
60	40	0.016780	0.016780	0.	0.010350	0.010350	0.	0.005366	0.005366	0.
60	50	0.038920	0.038920	0.	0.033610	0.033610	0.	0.028360	0.028360	0.
60	60	0.066990	0.066990	0.	0.066990	0.066990	0.	0.066990	0.066990	0.
60	70	0.088110	0.088110	0.	0.092360	0.092360	0.	0.096710	0.096710	0.
60	80	0.096420	0.096420	0.	0.101700	0.101700	0.	0.106900	0.106900	0.
60	90	0.097290	0.097290	0.	0.102400	0.102400	0.	0.107500	0.107500	0.
70	10	0.	0.	0.	0.	0.	0.	0.	0.	0.
70	20	0.	0.	0.	0.	0.	0.	0.	0.	0.
70	30	0.000014	0.000014	0.	0.	0.	0.	0.	0.	0.
70	40	0.001856	0.001856	0.	0.000546	0.000546	0.	0.000046	0.000046	0.
70	50	0.008186	0.008186	0.	0.005786	0.005786	0.	0.003856	0.003856	0.
70	60	0.018490	0.018490	0.	0.016690	0.016690	0.	0.015000	0.015000	0.
70	70	0.030150	0.030150	0.	0.030150	0.030150	0.	0.030150	0.030150	0.
70	80	0.037440	0.037440	0.	0.038620	0.038620	0.	0.039750	0.039750	0.
70	90	0.038790	0.038790	0.	0.040080	0.040080	0.	0.041300	0.041300	0.
80	10	0.	0.	0.	0.	0.	0.	0.	0.	0.
80	20	0.	0.	0.	0.	0.	0.	0.	0.	0.
80	30	0.	0.	0.	0.	0.	0.	0.	0.	0.
80	40	0.	0.	0.	0.	0.	0.	0.	0.	0.
80	50	0.000159	0.000159	0.	0.000019	0.000019	0.	0.	0.	0.
80	60	0.001736	0.001736	0.	0.001253	0.001253	0.	0.000879	0.000879	0.
80	70	0.004648	0.004648	0.	0.004280	0.004280	0.	0.003950	0.003950	0.
80	80	0.007596	0.007596	0.	0.007596	0.007596	0.	0.007596	0.007596	0.
80	90	0.008630	0.008630	0.	0.008768	0.008768	0.	0.008894	0.008894	0.

TABLE IV. - VIEW FACTOR $F_{B_1-B_2}$ BETWEEN EQUATORIAL TOROIDAL BANDS AND BANDS ABOVE AND BELOW TOROID EQUATOR

α_1	α_2	ρ											
		0.01			0.10			0.20			0.30		
		Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band
10	10	0.001724	0.000864	0.000860	0.018529	0.009427	0.009102	0.040812	0.020990	0.019822	0.068385	0.035561	0.032824
10	20	0.003382	0.001698	0.001684	0.035692	0.018249	0.017443	0.077576	0.040202	0.037373	0.128118	0.067380	0.060738
10	30	0.004913	0.002470	0.002443	0.050884	0.026064	0.024820	0.108910	0.056639	0.052271	0.176910	0.093532	0.083378
10	40	0.006270	0.003155	0.003114	0.063759	0.032702	0.031057	0.134340	0.070026	0.064314	0.214608	0.113876	0.100731
10	50	0.007416	0.003733	0.003683	0.074080	0.038031	0.036049	0.153698	0.080266	0.073432	0.241635	0.128590	0.113046
10	60	0.008327	0.004192	0.004135	0.081718	0.041989	0.039729	0.167075	0.087395	0.079680	0.258836	0.138083	0.120753
10	70	0.008979	0.004521	0.004458	0.086652	0.044560	0.042093	0.174820	0.091582	0.083239	0.267436	0.142975	0.124461
10	80	0.009358	0.004713	0.004646	0.089011	0.045807	0.043204	0.177667	0.093188	0.084478	0.269449	0.144251	0.125198
10	90	0.009468	0.004769	0.004700	0.089362	0.046004	0.043358	0.177799	0.093281	0.084518	0.269453	0.144254	0.125198
20	10	0.001691	0.000849	0.000842	0.017816	0.009109	0.008707	0.038642	0.020025	0.018617	0.063648	0.033475	0.030173
20	20	0.003318	0.001672	0.001646	0.034617	0.017936	0.016681	0.074465	0.039423	0.035042	0.121573	0.065890	0.055683
20	30	0.004823	0.002437	0.002386	0.049557	0.025838	0.023719	0.105248	0.056306	0.048941	0.169501	0.093250	0.076251
20	40	0.006160	0.003118	0.003042	0.062223	0.032559	0.029663	0.130256	0.070120	0.060136	0.206615	0.114701	0.091914
20	50	0.007290	0.003692	0.003598	0.072380	0.037970	0.034411	0.149313	0.080740	0.068573	0.233309	0.130370	0.102939
20	60	0.008187	0.004148	0.004040	0.079901	0.042000	0.037901	0.162508	0.088192	0.074316	0.250383	0.140638	0.109745
20	70	0.008829	0.004474	0.004355	0.084765	0.044635	0.040131	0.170177	0.092638	0.077540	0.259021	0.146105	0.112917
20	80	0.009202	0.004665	0.004537	0.087096	0.045933	0.041164	0.173034	0.094423	0.078611	0.261192	0.147718	0.113474
20	90	0.009311	0.004721	0.004590	0.087450	0.046152	0.041298	0.173194	0.094556	0.078637	0.261215	0.147741	0.113474
30	10	0.001637	0.000823	0.000814	0.016886	0.008650	0.008237	0.035944	0.018694	0.017250	0.057978	0.030654	0.027324
30	20	0.003215	0.001624	0.001590	0.032947	0.017178	0.015769	0.069734	0.037308	0.032425	0.111818	0.061517	0.050301
30	30	0.004678	0.002373	0.002305	0.047429	0.025020	0.022409	0.099454	0.054225	0.045229	0.157937	0.089209	0.068728
30	40	0.005981	0.003040	0.002940	0.059750	0.031740	0.028010	0.123758	0.068251	0.055507	0.194067	0.111383	0.082684
30	50	0.007084	0.003605	0.003478	0.069636	0.037161	0.032475	0.142317	0.079093	0.063224	0.220176	0.127738	0.092438
30	60	0.007958	0.004053	0.003905	0.076963	0.041213	0.035750	0.155204	0.086760	0.068444	0.237006	0.138615	0.098392
30	70	0.008583	0.004374	0.004209	0.081710	0.043878	0.037832	0.162739	0.091401	0.071338	0.245670	0.144583	0.101087
30	80	0.008947	0.004562	0.004385	0.083995	0.045210	0.038785	0.165604	0.093344	0.072260	0.248053	0.146538	0.101514
30	90	0.009053	0.004618	0.004435	0.084353	0.045451	0.038902	0.165804	0.093526	0.072278	0.248120	0.146606	0.101514
40	10	0.001566	0.000788	0.000778	0.015809	0.008109	0.007701	0.032974	0.017189	0.015785	0.052001	0.027595	0.024406
40	20	0.003078	0.001558	0.001520	0.030909	0.016174	0.014735	0.064184	0.034553	0.029631	0.100776	0.055946	0.044830
40	30	0.004484	0.002280	0.002204	0.044643	0.023715	0.020928	0.092040	0.050759	0.041281	0.143486	0.082353	0.061133
40	40	0.005738	0.002926	0.002812	0.056461	0.030314	0.026146	0.115277	0.064670	0.050607	0.177993	0.104571	0.073422
40	50	0.006801	0.003475	0.003326	0.065974	0.035674	0.030300	0.133137	0.075550	0.057587	0.203239	0.121274	0.081965
40	60	0.007645	0.003910	0.003734	0.073032	0.039691	0.033341	0.145586	0.083300	0.062286	0.219666	0.132535	0.087132
40	70	0.008248	0.004223	0.004025	0.077616	0.042348	0.035267	0.152920	0.088056	0.064864	0.228297	0.138880	0.089417
40	80	0.008599	0.004406	0.004193	0.079834	0.043696	0.036138	0.155777	0.090122	0.065655	0.230898	0.141145	0.089752
40	90	0.008701	0.004460	0.004241	0.080194	0.043954	0.036241	0.156024	0.090356	0.065668	0.231032	0.141280	0.089752

50	10	0.001481	0.000746	0.000736	0.014625	0.007508	0.007117	0.029866	0.015598	0.014269	0.046022	0.024492	0.021529
50	20	0.002913	0.001475	0.001438	0.028628	0.015018	0.013610	0.058248	0.031498	0.026750	0.089446	0.049982	0.039464
50	30	0.004247	0.002162	0.002085	0.041429	0.022109	0.019320	0.083793	0.046568	0.037225	0.127956	0.074235	0.053721
50	40	0.005439	0.002779	0.002660	0.052531	0.028405	0.024126	0.105401	0.059811	0.045590	0.159749	0.095323	0.064426
50	50	0.006451	0.003305	0.003147	0.061556	0.033608	0.027948	0.122309	0.070474	0.051835	0.183697	0.111859	0.071838
50	60	0.007255	0.003723	0.003533	0.068274	0.037532	0.030742	0.134186	0.078162	0.056024	0.199525	0.123232	0.076293
50	70	0.007830	0.004023	0.003808	0.072647	0.040142	0.032506	0.141240	0.082935	0.058305	0.208017	0.129787	0.078231
50	80	0.008165	0.004199	0.003966	0.074778	0.041481	0.033297	0.144061	0.085076	0.058985	0.210793	0.132293	0.078500
50	90	0.008263	0.004252	0.004011	0.075137	0.041751	0.033386	0.144354	0.085358	0.058996	0.211008	0.132507	0.078500
60	10	0.001386	0.000698	0.000688	0.013370	0.006870	0.006500	0.026726	0.013982	0.012744	0.040252	0.021477	0.018775
60	20	0.002725	0.001380	0.001345	0.026190	0.013766	0.012423	0.052187	0.028324	0.023863	0.078378	0.044028	0.034350
60	30	0.003974	0.002024	0.001950	0.037945	0.020319	0.017626	0.075224	0.042051	0.033172	0.112457	0.065774	0.046684
60	40	0.005092	0.002604	0.002487	0.048191	0.026191	0.022000	0.094877	0.054286	0.040591	0.140967	0.085051	0.055915
60	50	0.006043	0.003101	0.002942	0.056580	0.031103	0.025476	0.110458	0.064341	0.046118	0.162895	0.100607	0.062288
60	60	0.006800	0.003497	0.003303	0.062880	0.034865	0.028015	0.121595	0.071778	0.049817	0.177825	0.111720	0.066105
60	70	0.007342	0.003781	0.003560	0.066999	0.037384	0.029615	0.128284	0.076462	0.051822	0.186037	0.118288	0.067749
60	80	0.007657	0.003949	0.003708	0.069019	0.038691	0.030328	0.131026	0.078618	0.052409	0.188905	0.120934	0.067971
60	90	0.007750	0.004000	0.003750	0.069373	0.038966	0.030406	0.131357	0.078939	0.052417	0.189197	0.121226	0.067971
70	10	0.001280	0.000644	0.000635	0.012076	0.006210	0.005866	0.023642	0.012386	0.011256	0.034843	0.018629	0.016213
70	20	0.002517	0.001276	0.001242	0.023666	0.012462	0.011204	0.046202	0.025152	0.021050	0.067929	0.038319	0.029611
70	30	0.003672	0.001871	0.001800	0.034314	0.018427	0.015887	0.066684	0.037453	0.029231	0.097663	0.057480	0.040184
70	40	0.004706	0.002409	0.002297	0.043624	0.023802	0.019821	0.084253	0.048516	0.035737	0.122747	0.074671	0.048076
70	50	0.005587	0.002870	0.002717	0.051280	0.028336	0.022945	0.098294	0.057717	0.040577	0.142287	0.088776	0.053512
70	60	0.006289	0.003239	0.003050	0.057067	0.031843	0.025225	0.108454	0.064642	0.043812	0.155862	0.099101	0.056762
70	70	0.006792	0.003505	0.003287	0.060884	0.034223	0.026661	0.114670	0.069109	0.045562	0.163589	0.105431	0.058158
70	80	0.007086	0.003662	0.003424	0.062770	0.035471	0.027299	0.117282	0.071213	0.046069	0.166440	0.108095	0.058344
70	90	0.007172	0.003710	0.003462	0.063112	0.035744	0.027368	0.117633	0.071557	0.046076	0.166789	0.108445	0.058344
80	10	0.001166	0.000587	0.000579	0.010779	0.005548	0.005232	0.020713	0.010866	0.009847	0.029980	0.016052	0.013928
80	20	0.002294	0.001163	0.001131	0.021131	0.011144	0.009987	0.040497	0.022100	0.018397	0.058491	0.033080	0.025411
80	30	0.003347	0.001707	0.001640	0.030653	0.016499	0.014154	0.058494	0.032970	0.025524	0.084193	0.049737	0.034456
80	40	0.004290	0.002198	0.002092	0.038993	0.021342	0.017651	0.073985	0.042801	0.031183	0.105988	0.064789	0.041198
80	50	0.005094	0.002620	0.002475	0.045870	0.025445	0.020425	0.086428	0.051040	0.035389	0.123100	0.077261	0.045839
80	60	0.005736	0.002958	0.002778	0.051087	0.028639	0.022449	0.095492	0.057294	0.038198	0.135120	0.085509	0.048612
80	70	0.006196	0.003202	0.002994	0.054547	0.030824	0.023723	0.101102	0.061386	0.039716	0.142092	0.092289	0.049803
80	80	0.006465	0.003347	0.003119	0.056274	0.031985	0.024289	0.103518	0.063363	0.040155	0.144790	0.094828	0.049962
80	90	0.006545	0.003391	0.003154	0.056595	0.032245	0.024349	0.103869	0.063707	0.040162	0.145161	0.095200	0.049962
90	10	0.001049	0.000528	0.000520	0.009551	0.004918	0.004633	0.018121	0.009508	0.008612	0.025968	0.013901	0.012066
90	20	0.002063	0.001046	0.001017	0.018726	0.009884	0.008841	0.035441	0.019351	0.016090	0.050660	0.028644	0.022016
90	30	0.003009	0.001535	0.001474	0.027168	0.014642	0.012526	0.051217	0.028891	0.022326	0.072935	0.043080	0.029855
90	40	0.003857	0.001977	0.001880	0.034569	0.018952	0.015617	0.064812	0.037534	0.027278	0.091850	0.056150	0.035700
90	50	0.004581	0.002357	0.002224	0.040676	0.022609	0.018068	0.075752	0.044794	0.030958	0.106732	0.067007	0.039724
90	60	0.005158	0.002662	0.002496	0.045317	0.025462	0.019855	0.083745	0.050329	0.033416	0.117227	0.075097	0.042130
90	70	0.005573	0.002882	0.002691	0.048401	0.027421	0.020980	0.088710	0.053965	0.034745	0.123353	0.080190	0.043163
90	80	0.005815	0.003013	0.002803	0.049946	0.028466	0.021480	0.090865	0.055736	0.035129	0.125752	0.082452	0.043301
90	90	0.005887	0.003053	0.002834	0.050236	0.028703	0.021533	0.091188	0.056053	0.035135	0.126101	0.082800	0.043301

TABLE IV. - Continued. VIEW FACTOR $F_{B_1-B_2}$ BETWEEN EQUATORIAL TOROIDAL BANDS AND BANDS ABOVE AND BELOW TOROID EQUATOR

α_1	α_2	ρ											
		0.40			0.50			0.60			0.70		
		Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band
10	10	0.103462	0.054545	0.048918	0.149570	0.080309	0.069260	0.212626	0.117243	0.095383	0.303076	0.174483	0.128593
10	20	0.190446	0.101987	0.088460	0.268982	0.147560	0.121422	0.369952	0.210114	0.159838	0.501340	0.300594	0.200745
10	30	0.257788	0.139095	0.118693	0.354892	0.196732	0.158160	0.471524	0.271549	0.199975	0.608913	0.371368	0.237545
10	40	0.306675	0.166335	0.140341	0.412206	0.230096	0.182110	0.531501	0.308728	0.222772	0.661446	0.407142	0.254304
10	50	0.339146	0.184688	0.154458	0.446483	0.250509	0.195973	0.562169	0.328463	0.233707	0.681923	0.422107	0.259815
10	60	0.357641	0.195401	0.162239	0.462999	0.260801	0.202198	0.573120	0.336211	0.236909	0.685638	0.425402	0.260237
10	70	0.364975	0.199929	0.165046	0.467197	0.263822	0.203375	0.574187	0.337190	0.236998	0.685665	0.425429	0.260237
10	80	0.365711	0.200500	0.165211	0.467260	0.263885	0.203375	0.574187	0.337190	0.236998	0.685665	0.425429	0.260237
10	90	0.365711	0.200500	0.165211	0.467260	0.263885	0.203375	0.574187	0.337190	0.236998	0.685665	0.425429	0.260237
20	10	0.094277	0.050489	0.043788	0.132498	0.072688	0.059810	0.180907	0.102748	0.078159	0.242205	0.145212	0.096992
20	20	0.178205	0.099365	0.078840	0.247205	0.143039	0.104165	0.331958	0.202188	0.129770	0.435666	0.285929	0.149736
20	30	0.244494	0.139109	0.105385	0.332469	0.197506	0.134963	0.435286	0.273996	0.161290	0.553229	0.377196	0.175033
20	40	0.292879	0.168681	0.124198	0.390059	0.235319	0.154739	0.498007	0.319192	0.178815	0.614318	0.425682	0.187636
20	50	0.325255	0.188954	0.136301	0.425089	0.259156	0.165934	0.531391	0.344501	0.186891	0.640902	0.449803	0.191100
20	60	0.343935	0.201131	0.142803	0.442553	0.271849	0.170703	0.544673	0.355708	0.188966	0.648473	0.457165	0.191308
20	70	0.351626	0.206653	0.144974	0.447740	0.276286	0.171455	0.547143	0.358134	0.189010	0.649134	0.457826	0.191308
20	80	0.352686	0.207620	0.145067	0.448052	0.276598	0.171455	0.547172	0.358162	0.189010	0.649134	0.457826	0.191308
20	90	0.352687	0.207620	0.145067	0.448052	0.276598	0.171455	0.547172	0.358162	0.189010	0.649134	0.457826	0.191308
30	10	0.083701	0.045163	0.038538	0.113760	0.063059	0.050701	0.148323	0.085411	0.062912	0.185841	0.113346	0.072495
30	20	0.160375	0.091251	0.069125	0.216379	0.128546	0.087833	0.280067	0.176300	0.103767	0.349441	0.238251	0.111190
30	30	0.224082	0.131982	0.092101	0.298667	0.185338	0.113329	0.381585	0.253201	0.128384	0.470939	0.340716	0.130223
30	40	0.271516	0.163268	0.108248	0.356224	0.226711	0.129513	0.447257	0.305386	0.141872	0.542127	0.403568	0.138460
30	50	0.303602	0.185081	0.118522	0.392072	0.253554	0.138518	0.484032	0.336124	0.147908	0.576818	0.436037	0.140781
30	60	0.322461	0.198537	0.123924	0.410757	0.268553	0.142194	0.500448	0.351115	0.149332	0.589861	0.448949	0.140912
30	70	0.330632	0.205021	0.125611	0.417267	0.274560	0.142707	0.505005	0.355645	0.149360	0.592565	0.451653	0.140912
30	80	0.332169	0.206497	0.125672	0.418083	0.275376	0.142707	0.505326	0.355965	0.149360	0.592628	0.451716	0.140912
30	90	0.332180	0.206508	0.125672	0.418084	0.275377	0.142707	0.505326	0.355965	0.149360	0.592628	0.451716	0.140912
40	10	0.073063	0.039628	0.033435	0.095957	0.053560	0.042397	0.119640	0.069487	0.050153	0.141215	0.086928	0.054287
40	20	0.140956	0.081181	0.059775	0.184359	0.111225	0.073134	0.229298	0.146972	0.082326	0.271397	0.188495	0.082902
40	30	0.199223	0.119796	0.079427	0.258709	0.164652	0.094056	0.320067	0.218542	0.101525	0.379232	0.282381	0.096851
40	40	0.244695	0.151547	0.093148	0.314607	0.207379	0.107228	0.385868	0.273921	0.111947	0.455104	0.352273	0.102831
40	50	0.276154	0.174348	0.101806	0.350873	0.236401	0.114472	0.425516	0.308984	0.116532	0.497267	0.392791	0.104477
40	60	0.295039	0.188753	0.106287	0.370660	0.253318	0.117342	0.444989	0.327422	0.117567	0.516042	0.411474	0.104569
40	70	0.303673	0.196056	0.107617	0.378507	0.260790	0.117717	0.451850	0.334263	0.117587	0.521838	0.417269	0.104569
40	80	0.305734	0.198072	0.107662	0.380037	0.262320	0.117717	0.452888	0.335301	0.117587	0.522451	0.417883	0.104569
40	90	0.305794	0.198132	0.107662	0.380055	0.262338	0.117717	0.452890	0.335303	0.117587	0.522451	0.417883	0.104569

50	10	0.062929	0.034269	0.028660	0.079971	0.044866	0.035105	0.095730	0.055928	0.039803	0.107378	0.066463	0.040916
50	20	0.121916	0.070824	0.051092	0.154571	0.094229	0.060342	0.185089	0.119997	0.065092	0.208900	0.146613	0.062288
50	30	0.173497	0.105764	0.067732	0.219068	0.141667	0.077401	0.262036	0.181965	0.080071	0.297638	0.224991	0.072647
50	40	0.215077	0.135786	0.079291	0.269960	0.181883	0.088078	0.321899	0.233743	0.088156	0.366819	0.289746	0.077073
50	50	0.245190	0.158645	0.086545	0.305409	0.211498	0.093911	0.362162	0.270473	0.091689	0.412283	0.333994	0.078289
50	60	0.263813	0.173552	0.090261	0.325828	0.229642	0.096185	0.383867	0.291392	0.092475	0.435579	0.357222	0.078357
50	70	0.272757	0.181427	0.091331	0.334747	0.238274	0.096474	0.392736	0.300246	0.092490	0.444474	0.366117	0.078357
50	80	0.275288	0.183923	0.091366	0.337023	0.240549	0.096474	0.394752	0.302262	0.092490	0.446227	0.367870	0.078357
50	90	0.275437	0.184071	0.091366	0.337117	0.240644	0.096474	0.394805	0.302315	0.092490	0.446251	0.367894	0.078357
60	10	0.053620	0.029296	0.024325	0.066112	0.037237	0.028875	0.076406	0.044815	0.031590	0.082372	0.051115	0.031257
60	20	0.104168	0.060915	0.043253	0.128290	0.078801	0.049489	0.148506	0.096975	0.051531	0.161250	0.113685	0.047565
60	30	0.148896	0.091672	0.057224	0.182970	0.119626	0.063344	0.212096	0.148802	0.063294	0.232196	0.175727	0.055469
60	40	0.185668	0.118780	0.066888	0.227353	0.155374	0.071979	0.263531	0.193902	0.069629	0.290403	0.231555	0.058848
60	50	0.213162	0.140230	0.072933	0.259754	0.183071	0.076683	0.300506	0.228111	0.072394	0.332295	0.272520	0.059775
60	60	0.230997	0.154982	0.076015	0.279865	0.201356	0.078509	0.322779	0.249770	0.073009	0.357333	0.297506	0.059827
60	70	0.239995	0.163105	0.076890	0.289397	0.210658	0.078739	0.332972	0.259951	0.073021	0.368454	0.308627	0.059827
60	80	0.242862	0.165944	0.076918	0.292276	0.213537	0.078739	0.335886	0.262865	0.073021	0.371444	0.311617	0.059827
60	90	0.243117	0.166199	0.076918	0.292498	0.213759	0.078739	0.336077	0.263056	0.073021	0.371605	0.311778	0.059827
70	10	0.045341	0.024838	0.020503	0.054498	0.030770	0.023728	0.061371	0.036040	0.025331	0.064398	0.039943	0.024454
70	20	0.088243	0.051862	0.036381	0.106026	0.065424	0.040603	0.119634	0.078310	0.041323	0.126232	0.089029	0.037203
70	30	0.126501	0.078442	0.048059	0.151831	0.099908	0.051923	0.171646	0.120887	0.050759	0.182418	0.139040	0.043378
70	40	0.158345	0.102229	0.056117	0.189650	0.130672	0.058978	0.214614	0.158770	0.055844	0.229664	0.183644	0.046020
70	50	0.182604	0.121457	0.061147	0.217987	0.155167	0.062821	0.246569	0.188505	0.058064	0.265196	0.218451	0.046745
70	60	0.198838	0.135128	0.063710	0.236389	0.172076	0.064313	0.267005	0.208448	0.058557	0.288093	0.241307	0.046786
70	70	0.207509	0.143074	0.064436	0.245865	0.181364	0.064501	0.277479	0.218913	0.058566	0.299895	0.253109	0.046786
70	80	0.210524	0.146065	0.064459	0.249083	0.184583	0.064501	0.280961	0.222394	0.058566	0.303728	0.255942	0.046786
70	90	0.210873	0.146414	0.064459	0.249435	0.184934	0.064501	0.281318	0.222752	0.058566	0.304097	0.257311	0.046786
80	10	0.038327	0.021018	0.017309	0.045330	0.025589	0.019741	0.050160	0.029454	0.020705	0.051383	0.031883	0.019500
80	20	0.074676	0.043970	0.030706	0.088250	0.054464	0.033787	0.097712	0.063949	0.033764	0.100708	0.071034	0.029674
80	30	0.107235	0.066674	0.040562	0.126551	0.083341	0.043211	0.140266	0.098802	0.041465	0.145516	0.110912	0.034605
80	40	0.134521	0.087159	0.047362	0.158407	0.109328	0.049079	0.175689	0.130073	0.045617	0.183415	0.145703	0.036712
80	50	0.155523	0.103914	0.051608	0.182581	0.130307	0.052275	0.202447	0.155017	0.047430	0.212377	0.175087	0.037290
80	60	0.169801	0.116030	0.053771	0.198617	0.145102	0.053515	0.220042	0.172209	0.047833	0.231736	0.194413	0.037323
80	70	0.177649	0.123266	0.054384	0.207204	0.153533	0.053671	0.229506	0.181665	0.047841	0.242300	0.204977	0.037323
80	80	0.180575	0.126172	0.054404	0.210408	0.156736	0.053671	0.233054	0.185213	0.047841	0.246286	0.208963	0.037323
80	90	0.180973	0.126569	0.054404	0.210836	0.157165	0.053671	0.233522	0.185681	0.047841	0.246804	0.209481	0.037323
90	10	0.032836	0.018005	0.014831	0.038266	0.021614	0.016652	0.041590	0.024416	0.017173	0.041704	0.025867	0.015837
90	20	0.063962	0.037655	0.026307	0.074500	0.045999	0.028501	0.081079	0.053068	0.028011	0.081751	0.057657	0.024094
90	30	0.091817	0.057074	0.034742	0.106829	0.070372	0.036457	0.116412	0.082006	0.034406	0.118165	0.090072	0.028093
90	40	0.115174	0.074614	0.040560	0.133699	0.092288	0.041412	0.145768	0.107917	0.037851	0.148903	0.119099	0.029804
90	50	0.133191	0.089001	0.044191	0.154131	0.110021	0.044110	0.167940	0.128585	0.039355	0.172379	0.142105	0.030273
90	60	0.145508	0.099468	0.046040	0.167757	0.122599	0.045157	0.182620	0.142930	0.039689	0.188135	0.157835	0.030300
90	70	0.152340	0.105776	0.046564	0.175148	0.129859	0.045289	0.190649	0.150953	0.039696	0.196935	0.166635	0.030300
90	80	0.154931	0.108350	0.046581	0.177962	0.132673	0.045289	0.193732	0.154036	0.039696	0.200355	0.170056	0.030300
90	90	0.155309	0.108728	0.046581	0.178376	0.133087	0.045289	0.194190	0.154494	0.039696	0.200866	0.170566	0.030300

TABLE IV. - Concluded. VIEW FACTOR $F_{B_1-B_2}$ BETWEEN EQUATORIAL TOROIDAL BANDS AND BANDS ABOVE AND BELOW TOROID EQUATOR

α_1	α_2	ρ								
		0.80			0.90			0.99		
		Total	Upper band	Lower band	Total	Upper band	Lower band	Total	Upper band	Lower band
10	10	0.439322	0.273960	0.165361	0.647081	0.478967	0.168114	0.878952	0.860187	0.018764
10	20	0.669261	0.439927	0.229334	0.859659	0.666575	0.193083	0.982793	0.964028	0.018765
10	30	0.760508	0.508296	0.252212	0.901035	0.704094	0.196941	0.983590	0.964825	0.018765
10	40	0.792268	0.533410	0.258858	0.907123	0.710091	0.197032	0.983590	0.964825	0.018765
10	50	0.798868	0.539459	0.259409	0.907238	0.710206	0.197032	0.983590	0.964825	0.018765
10	60	0.799056	0.539646	0.259409	0.907238	0.710206	0.197032	0.983590	0.964825	0.018765
10	70	0.799056	0.539646	0.259409	0.907238	0.710206	0.197032	0.983590	0.964825	0.018765
10	80	0.799056	0.539646	0.259409	0.907238	0.710206	0.197032	0.983590	0.964825	0.018765
10	90	0.799056	0.539646	0.259409	0.907238	0.710206	0.197032	0.983590	0.964825	0.018765
20	10	0.315899	0.207646	0.108253	0.380441	0.295014	0.085427	0.245903	0.241196	0.004707
20	20	0.558837	0.409965	0.148873	0.692260	0.594131	0.098130	0.763051	0.758343	0.004708
20	30	0.682724	0.519629	0.163095	0.812405	0.712407	0.099998	0.899961	0.895253	0.004708
20	40	0.732968	0.566038	0.166930	0.843348	0.743310	0.100039	0.912169	0.907461	0.004708
20	50	0.749014	0.581820	0.167194	0.848972	0.748933	0.100039	0.912608	0.907901	0.004708
20	60	0.751678	0.584484	0.167194	0.849158	0.749119	0.100039	0.912608	0.907901	0.004708
20	70	0.751705	0.584511	0.167194	0.849158	0.749119	0.100039	0.912608	0.907901	0.004708
20	80	0.751705	0.584511	0.167194	0.849158	0.749119	0.100039	0.912608	0.907901	0.004708
20	90	0.751705	0.584511	0.167194	0.849158	0.749119	0.100039	0.912608	0.907901	0.004708
30	10	0.219150	0.146476	0.072674	0.223464	0.174630	0.048834	0.089946	0.088224	0.001722
30	20	0.416826	0.317266	0.099560	0.455430	0.399387	0.056043	0.327150	0.325428	0.001722
30	30	0.561472	0.452603	0.108869	0.639845	0.582749	0.057096	0.636661	0.634939	0.001722
30	40	0.636206	0.524908	0.111298	0.720840	0.663722	0.057118	0.750805	0.749082	0.001722
30	50	0.666815	0.555356	0.111459	0.746471	0.689353	0.057118	0.775112	0.773390	0.001722
30	60	0.676089	0.564630	0.111459	0.751809	0.694690	0.057118	0.778224	0.776502	0.001722
30	70	0.677239	0.565780	0.111459	0.752025	0.694906	0.057118	0.778228	0.776506	0.001722
30	80	0.677241	0.565782	0.111459	0.752025	0.694906	0.057118	0.778228	0.776506	0.001722
30	90	0.677241	0.565782	0.111459	0.752025	0.694906	0.057118	0.778228	0.776506	0.001722
40	10	0.153444	0.103313	0.050131	0.138415	0.108355	0.030059	0.041872	0.041074	0.000798
40	20	0.300734	0.232245	0.068489	0.290868	0.256373	0.034495	0.155678	0.154880	0.000798
40	30	0.427616	0.352811	0.074804	0.443502	0.408360	0.035142	0.355639	0.354841	0.000798
40	40	0.516398	0.439955	0.076443	0.556434	0.521277	0.035156	0.516346	0.515548	0.000798
40	50	0.561903	0.485352	0.076551	0.609156	0.574000	0.035156	0.591316	0.590518	0.000798
40	60	0.580034	0.503482	0.076551	0.627108	0.591951	0.035156	0.613182	0.612384	0.000798
40	70	0.584697	0.508145	0.076551	0.630619	0.595463	0.035156	0.617393	0.616595	0.000798
40	80	0.584983	0.508432	0.076551	0.630706	0.595550	0.035156	0.617435	0.616637	0.000798
40	90	0.584983	0.508432	0.076551	0.630706	0.595550	0.035156	0.617435	0.616637	0.000798

50	10	0.109505	0.073953	0.035552	0.090395	0.070771	0.019624	0.022458	0.022027	0.000431
50	20	0.217531	0.168983	0.048548	0.191185	0.168665	0.022520	0.083003	0.082572	0.000431
50	30	0.317209	0.264190	0.053018	0.299782	0.276839	0.022943	0.191792	0.191361	0.000431
50	40	0.397692	0.343514	0.054178	0.397574	0.374622	0.022952	0.327709	0.327278	0.000431
50	50	0.450658	0.396403	0.054255	0.464177	0.441225	0.022952	0.410253	0.409822	0.000431
50	60	0.476518	0.422263	0.054255	0.494822	0.471870	0.022952	0.451292	0.450861	0.000431
50	70	0.485603	0.431348	0.054255	0.504509	0.481557	0.022952	0.463440	0.463009	0.000431
50	80	0.487097	0.432842	0.054255	0.505773	0.482821	0.022952	0.465496	0.465065	0.000431
50	90	0.487106	0.432851	0.054255	0.505775	0.482823	0.022952	0.465496	0.465065	0.000431
60	10	0.080303	0.054232	0.026071	0.061816	0.048387	0.013429	0.013541	0.013282	0.000259
60	20	0.160062	0.124465	0.035597	0.130744	0.115334	0.015411	0.049807	0.049548	0.000259
60	30	0.235796	0.196921	0.038875	0.206714	0.191014	0.015700	0.116658	0.116399	0.000259
60	40	0.300964	0.261239	0.039725	0.280300	0.264594	0.015706	0.204353	0.204094	0.000259
60	50	0.349341	0.309560	0.039781	0.338583	0.322877	0.015706	0.276564	0.276305	0.000259
60	60	0.378480	0.338699	0.039781	0.374362	0.358656	0.015706	0.315992	0.315732	0.000259
60	70	0.390995	0.351214	0.039781	0.389170	0.373464	0.015706	0.334279	0.334020	0.000259
60	80	0.394130	0.354349	0.039781	0.392584	0.376878	0.015706	0.338979	0.338720	0.000259
60	90	0.394265	0.354484	0.039781	0.392695	0.376989	0.015706	0.339286	0.339027	0.000259
70	10	0.060560	0.040903	0.019657	0.044048	0.034486	0.009562	0.008833	0.008664	0.000169
70	20	0.120661	0.093820	0.026841	0.093193	0.082220	0.010974	0.032554	0.032385	0.000169
70	30	0.178131	0.148819	0.029312	0.147140	0.135960	0.011180	0.076053	0.075884	0.000169
70	40	0.228810	0.198857	0.029953	0.200584	0.189400	0.011184	0.130861	0.130692	0.000169
70	50	0.268524	0.238529	0.029995	0.245886	0.234702	0.011184	0.190242	0.190073	0.000169
70	60	0.294805	0.264810	0.029995	0.277321	0.266137	0.011184	0.221853	0.221684	0.000169
70	70	0.308449	0.278454	0.029995	0.293552	0.282368	0.011184	0.237442	0.237273	0.000169
70	80	0.312777	0.282782	0.029995	0.298613	0.287428	0.011184	0.242977	0.242808	0.000169
70	90	0.313165	0.283170	0.029995	0.299029	0.287845	0.011184	0.243783	0.243614	0.000169
80	10	0.046785	0.031598	0.015188	0.032474	0.025422	0.007052	0.006113	0.005996	0.000117
80	20	0.093208	0.072473	0.020734	0.068697	0.060605	0.008092	0.022583	0.022466	0.000117
80	30	0.137580	0.114936	0.022644	0.108538	0.100294	0.008244	0.052743	0.052626	0.000117
80	40	0.176824	0.153685	0.023139	0.147893	0.139646	0.008247	0.089971	0.089854	0.000117
80	50	0.207996	0.184824	0.023172	0.181910	0.173662	0.008247	0.133808	0.133691	0.000117
80	60	0.229590	0.206418	0.023172	0.206147	0.197900	0.008247	0.161373	0.161256	0.000117
80	70	0.241642	0.218470	0.023172	0.220403	0.212156	0.008247	0.172785	0.172668	0.000117
80	80	0.246180	0.223009	0.023172	0.225643	0.217396	0.008247	0.176279	0.176162	0.000117
80	90	0.246763	0.223591	0.023172	0.226309	0.218062	0.008247	0.177021	0.176904	0.000117
90	10	0.036925	0.024941	0.011984	0.024675	0.019315	0.005361	0.004438	0.004353	0.000085
90	20	0.073576	0.057211	0.016365	0.052190	0.046038	0.006152	0.016416	0.016331	0.000085
90	30	0.108611	0.090740	0.017872	0.082564	0.076297	0.006267	0.038410	0.038326	0.000085
90	40	0.139586	0.121323	0.018263	0.112481	0.106211	0.006270	0.064840	0.064755	0.000085
90	50	0.164147	0.145858	0.018289	0.137957	0.131687	0.006270	0.095248	0.095163	0.000085
90	60	0.181234	0.162945	0.018289	0.156730	0.150460	0.006270	0.121378	0.121293	0.000085
90	70	0.190935	0.172646	0.018289	0.167616	0.161346	0.006270	0.131996	0.131912	0.000085
90	80	0.194787	0.176498	0.018289	0.172238	0.165968	0.006270	0.134293	0.134208	0.000085
90	90	0.195362	0.177074	0.018289	0.172879	0.166610	0.006270	0.134295	0.134210	0.000085

TABLE V. - AREA OF EQUATORIAL TOROIDAL BANDS, $A_B(\alpha_a)/R^2$

α_a, deg	ρ										
	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
5.00	0.0054	0.0494	0.0878	0.1152	0.1317	0.1373	0.1318	0.1155	0.0882	0.0499	0.0061
10.00	0.0109	0.0988	0.1757	0.2308	0.2641	0.2755	0.2652	0.2330	0.1790	0.1032	0.0163
15.00	0.0163	0.1482	0.2639	0.3471	0.3978	0.4159	0.4015	0.3546	0.2752	0.1632	0.0346
20.00	0.0217	0.1978	0.3527	0.4646	0.5335	0.5594	0.5423	0.4823	0.3793	0.2333	0.0651
25.00	0.0272	0.2476	0.4421	0.5835	0.6718	0.7069	0.6890	0.6179	0.4938	0.3165	0.1116
30.00	0.0326	0.2976	0.5323	0.7042	0.8133	0.8595	0.8429	0.7635	0.6213	0.4162	0.1779
35.00	0.0380	0.3478	0.6235	0.8271	0.9586	1.0181	1.0055	0.9208	0.7641	0.5352	0.2676
40.00	0.0435	0.3983	0.7157	0.9525	1.1084	1.1836	1.1779	1.0916	0.9244	0.6765	0.3842
45.00	0.0489	0.4491	0.8092	1.0806	1.2631	1.3567	1.3614	1.2773	1.1044	0.8426	0.5310
50.00	0.0543	0.5002	0.9041	1.2117	1.4231	1.5383	1.5571	1.4797	1.3060	1.0361	0.7109
55.00	0.0598	0.5517	1.0004	1.3462	1.5891	1.7290	1.7660	1.7000	1.5311	1.2593	0.9266
60.00	0.0653	0.6036	1.0983	1.4842	1.7613	1.9295	1.9889	1.9395	1.7813	1.5142	1.1808
65.00	0.0707	0.6559	1.1978	1.6259	1.9401	2.1404	2.2268	2.1993	2.0580	1.8027	1.4756
70.00	0.0762	0.7086	1.2991	1.7715	2.1259	2.3621	2.4803	2.4804	2.3624	2.1263	1.8128
75.00	0.0816	0.7618	1.4022	1.9212	2.3188	2.5951	2.7499	2.7834	2.6955	2.4862	2.1941
80.00	0.0871	0.8154	1.5071	2.0750	2.5192	2.8396	3.0362	3.1091	3.0582	2.8836	2.6207
85.00	0.0926	0.8695	1.6139	2.2331	2.7270	3.0958	3.3394	3.4579	3.4511	3.3191	3.0934
90.00	0.0981	0.9241	1.7226	2.3954	2.9425	3.3640	3.6598	3.8300	3.8744	3.7933	3.6128
95.00	0.1036	0.9792	1.8332	2.5620	3.1657	3.6441	3.9974	4.2255	4.3284	4.3061	4.1790
100.00	0.1090	1.0347	1.9457	2.7330	3.3965	3.9362	4.3522	4.6443	4.8128	4.8575	4.7920
105.00	0.1145	1.0908	2.0601	2.9081	3.6348	4.2400	4.7239	5.0863	5.3274	5.4471	5.4511
110.00	0.1200	1.1472	2.1764	3.0875	3.8805	4.5554	5.1122	5.5509	5.8716	6.0741	6.1555
115.00	0.1255	1.2042	2.2945	3.2708	4.1333	4.8820	5.5167	6.0375	6.4444	6.7375	6.9039
120.00	0.1311	1.2615	2.4142	3.4581	4.3932	5.2194	5.9368	6.5453	7.0451	7.4360	7.6948
125.00	0.1366	1.3193	2.5357	3.6491	4.6596	5.5672	6.3718	7.0735	7.6722	8.1680	8.5262
130.00	0.1421	1.3775	2.6587	3.8436	4.9323	5.9247	6.8209	7.6208	8.3244	8.9318	9.3961
135.00	0.1476	1.4360	2.7832	4.0415	5.2109	6.2915	7.2832	8.1861	9.0001	9.7252	10.3019
140.00	0.1531	1.4949	2.9090	4.2423	5.4949	6.6667	7.7577	8.7679	9.6974	10.5461	11.2408
145.00	0.1586	1.5541	3.0360	4.4460	5.7838	7.0495	8.2432	9.3648	10.4143	11.3918	12.2098
150.00	0.1642	1.6135	3.1642	4.6521	6.0771	7.4393	8.7386	9.9752	11.1488	12.2597	13.2058
155.00	0.1697	1.6732	3.2933	4.8603	6.3742	7.8350	9.2427	10.5972	11.8987	13.1470	14.2251
160.00	0.1752	1.7331	3.4232	5.0704	6.6745	8.2357	9.7540	11.2292	12.6614	14.0507	15.2642
165.00	0.1808	1.7932	3.5538	5.2819	6.9775	8.6406	10.2711	11.8691	13.4346	14.9676	16.3194
170.00	0.1863	1.8534	3.6849	5.4946	7.2825	9.0485	10.7928	12.5152	14.2158	15.8946	17.3868
175.00	0.1919	1.9136	3.8163	5.7080	7.5887	9.4585	11.3173	13.1653	15.0022	16.8282	18.4623
180.00	0.1974	1.9739	3.9478	5.9218	7.8957	9.8696	11.8435	13.8174	15.7914	17.7653	19.5418

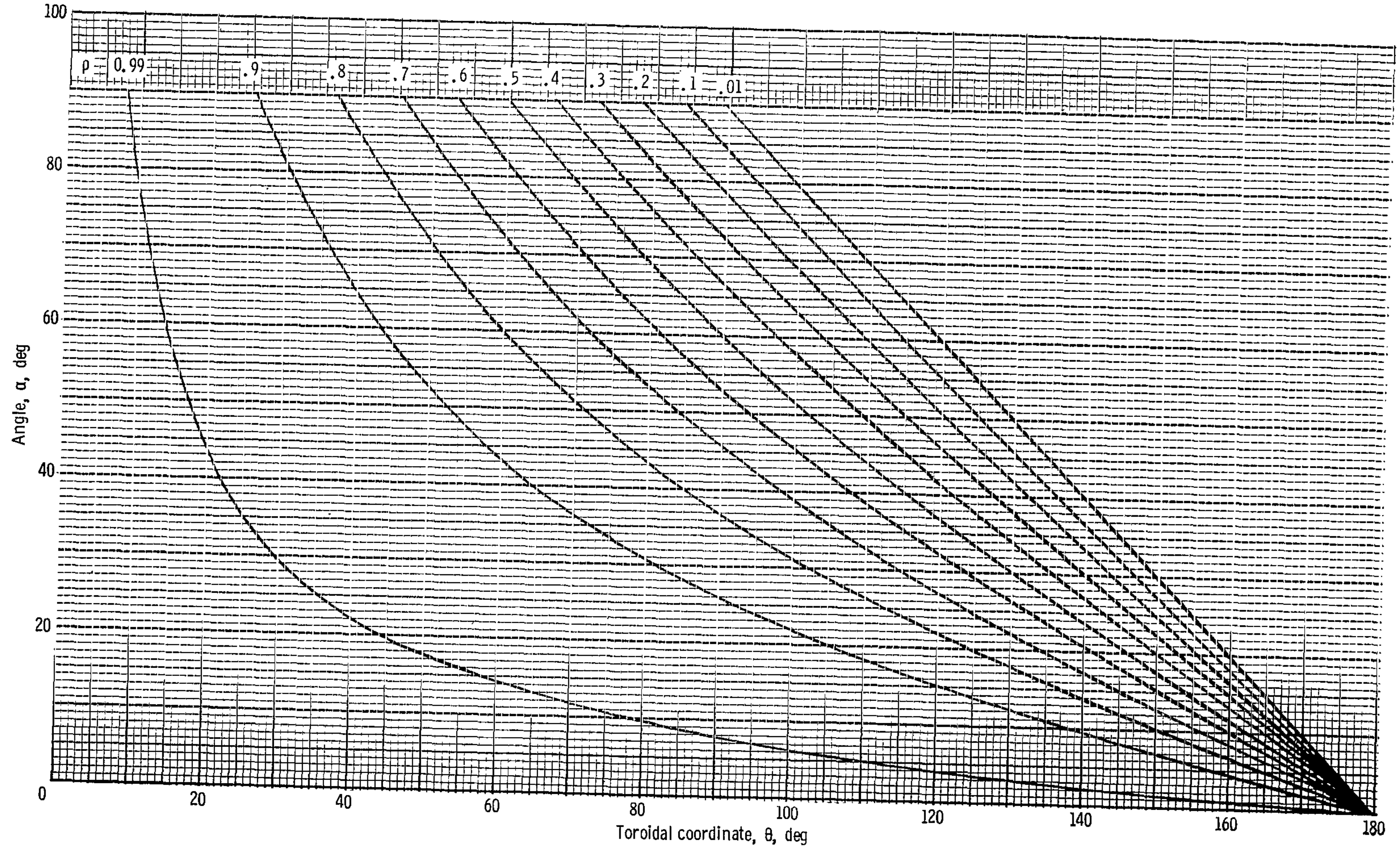
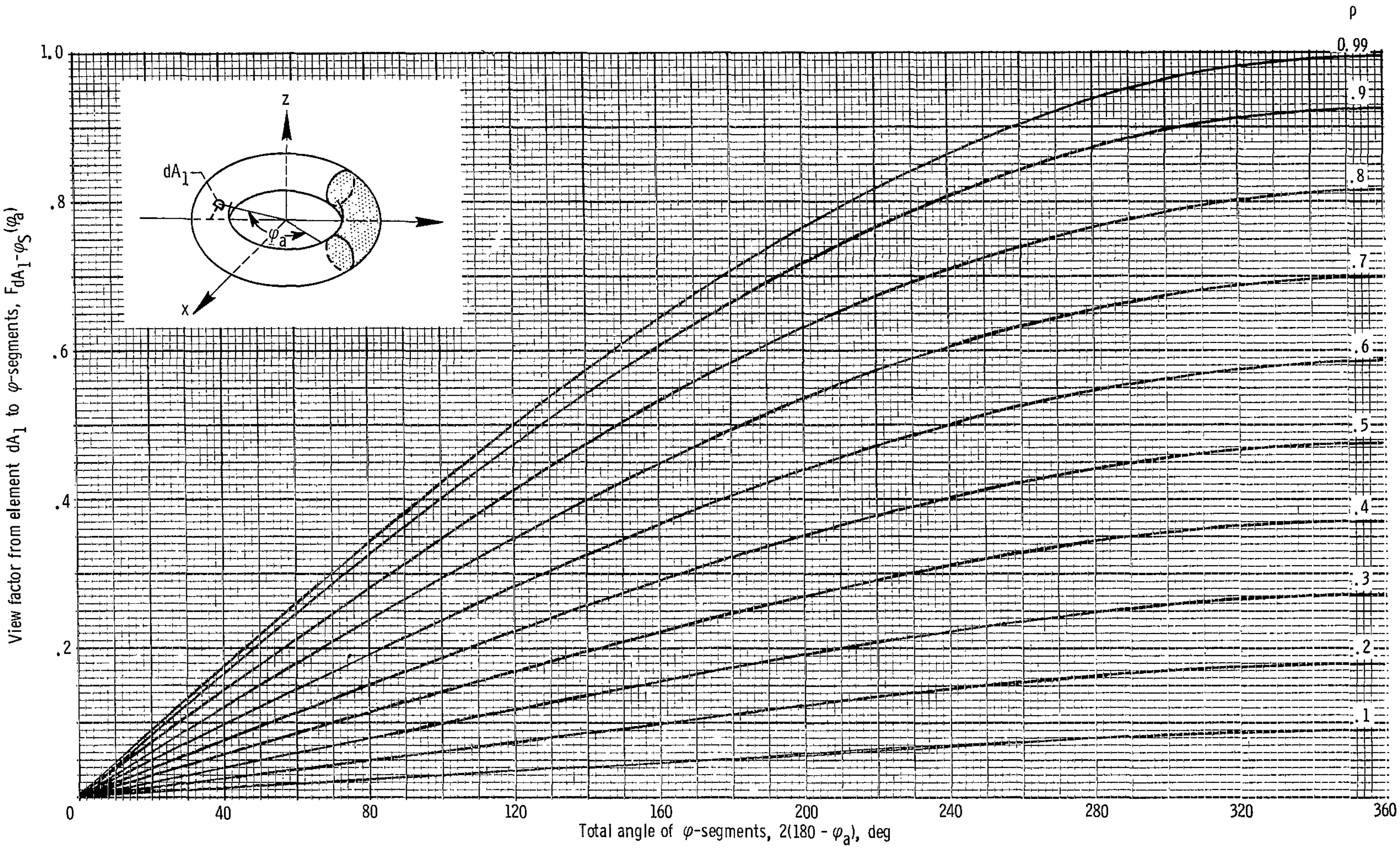
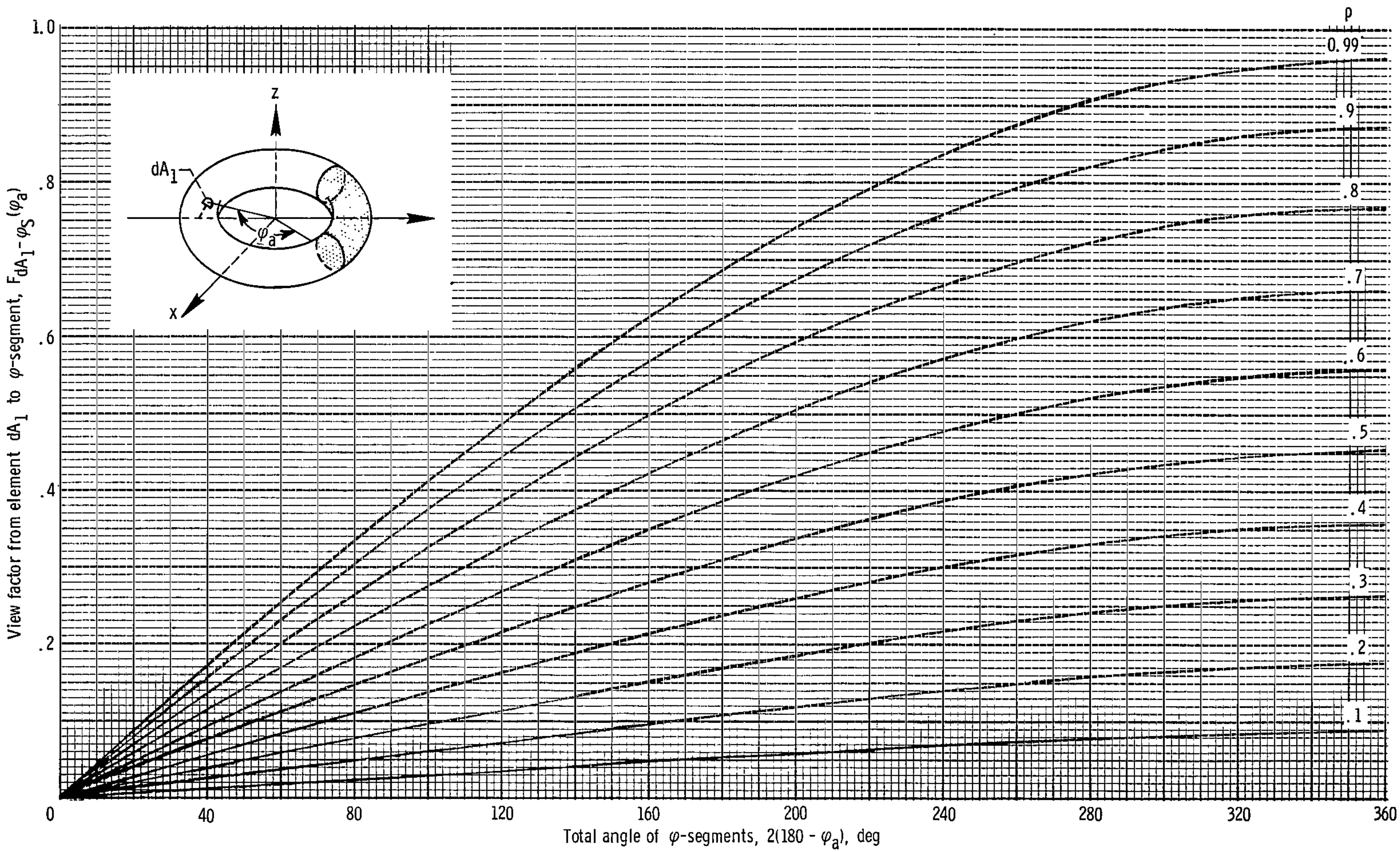


Figure 1. - Value for variable α as function of toroidal coordinate θ with ρ as a parameter.



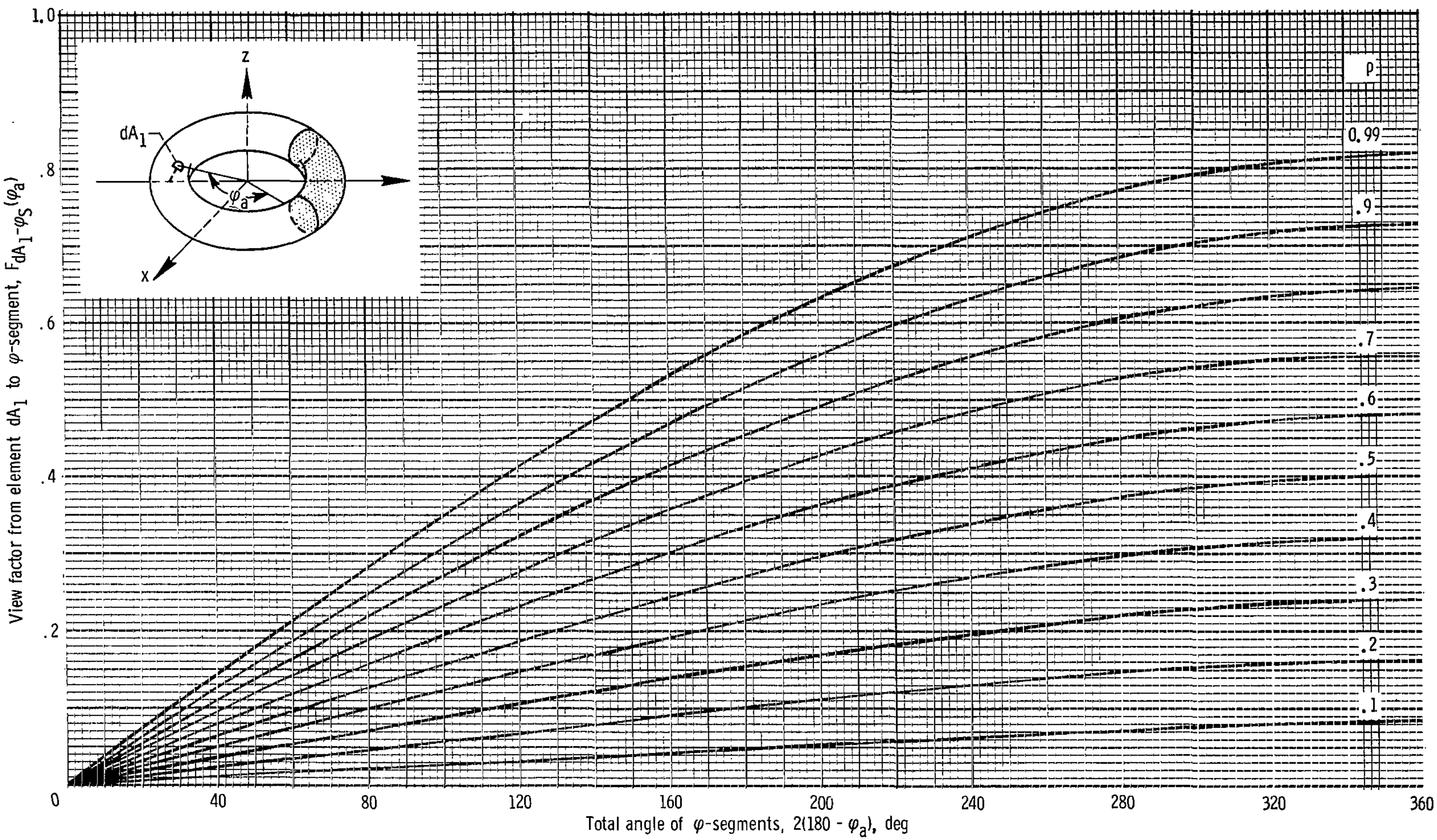
(a) $\alpha_1 = 0°$.

Figure 2. - View factor from element dA_1 to symmetrically located φ-segments about dA_1 as function of total subtended angle of the φ-segments.



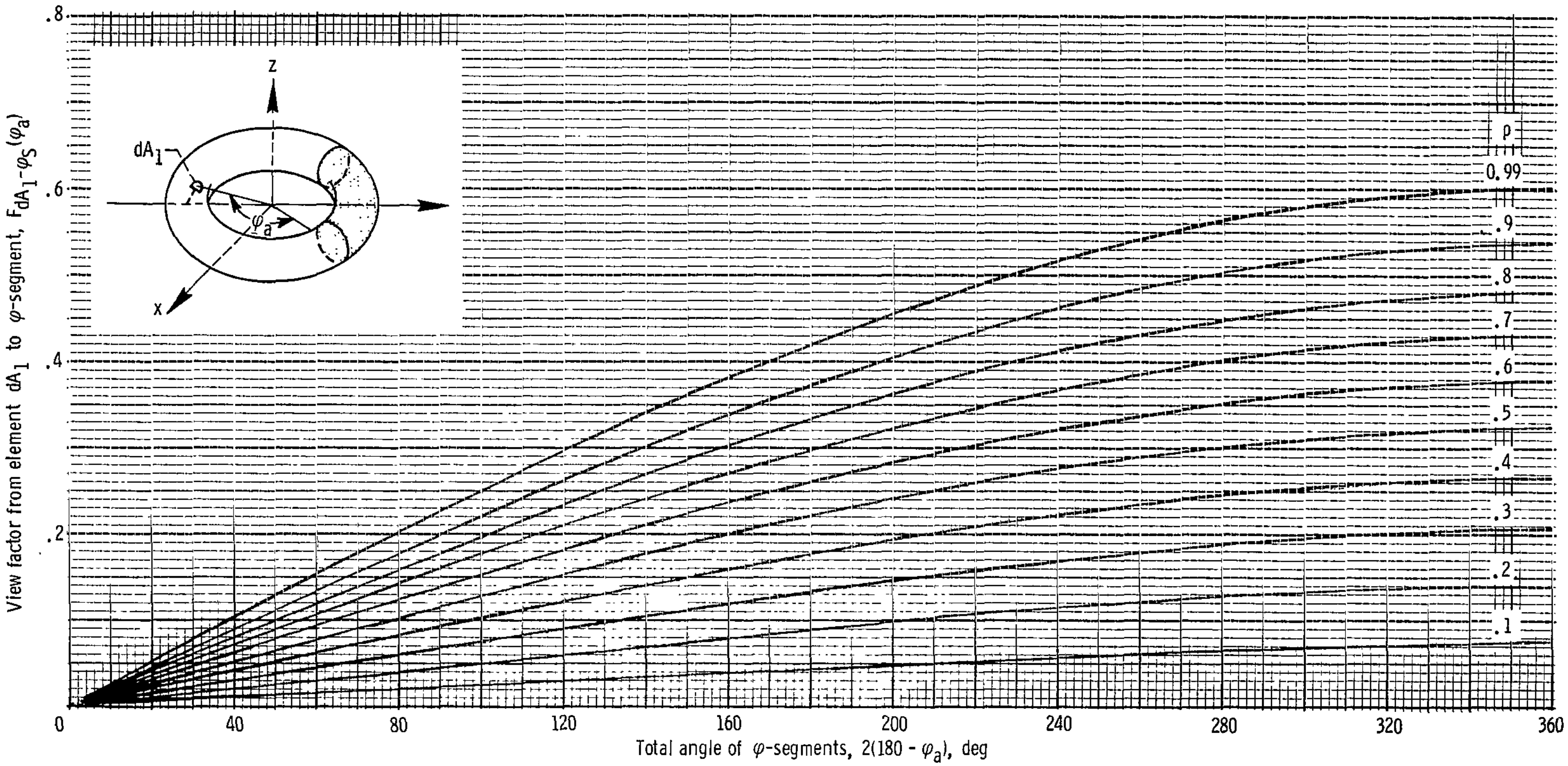
(b) $\alpha_1 = 10°$.

Figure 2. - Continued.



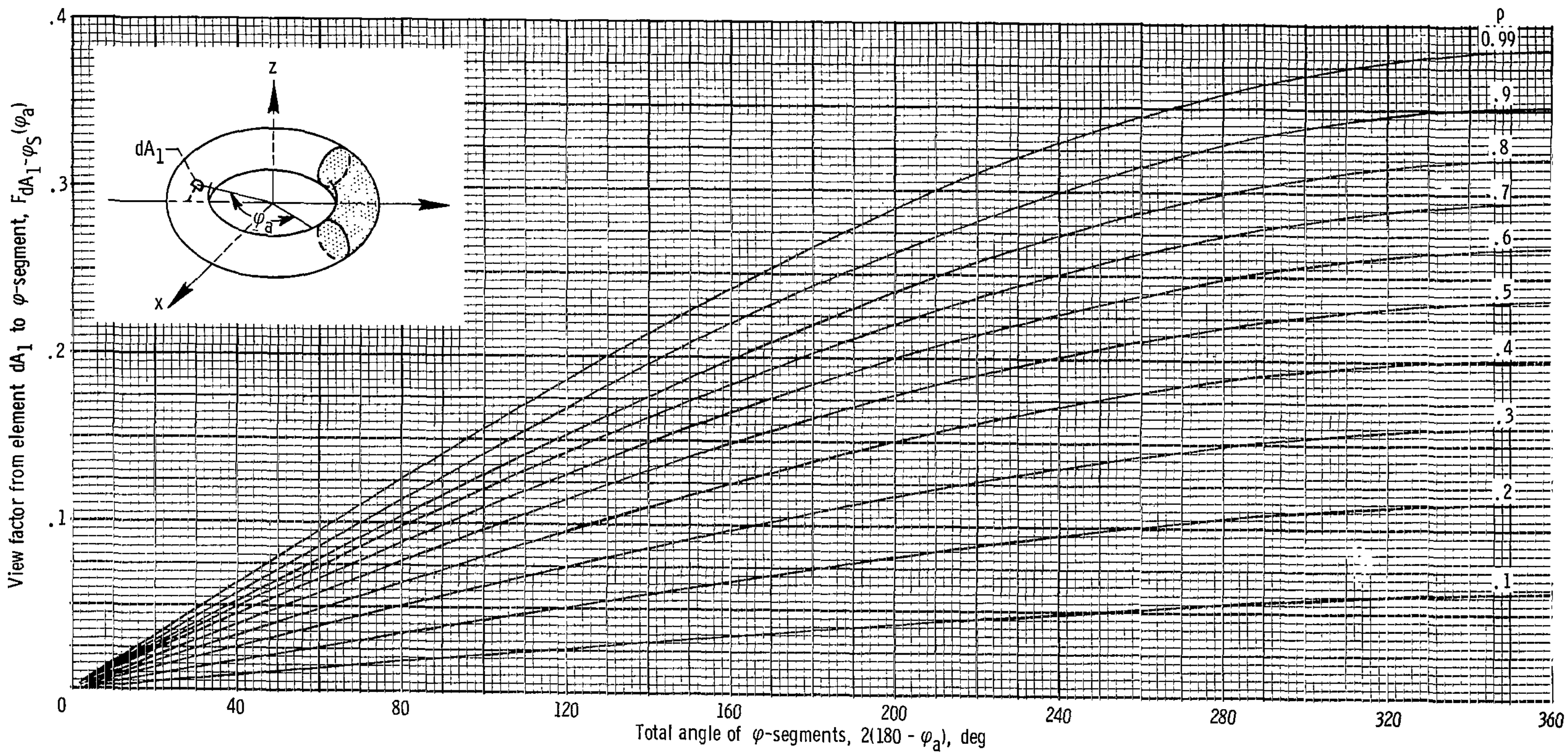
(c) $\alpha_1 = 20°$.

Figure 2. - Continued.



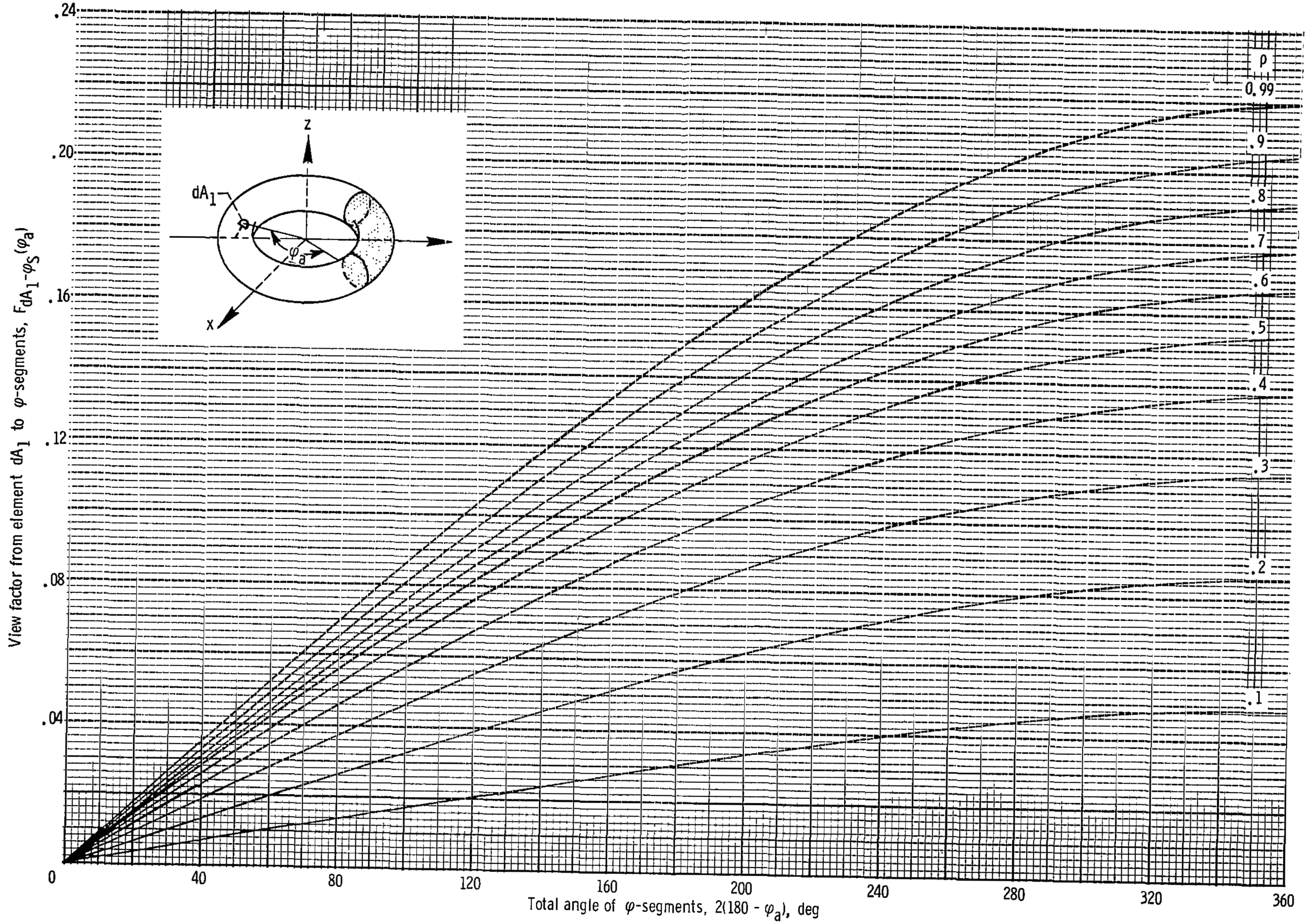
(d) $\alpha_1 = 30°$.

Figure 2. - Continued.



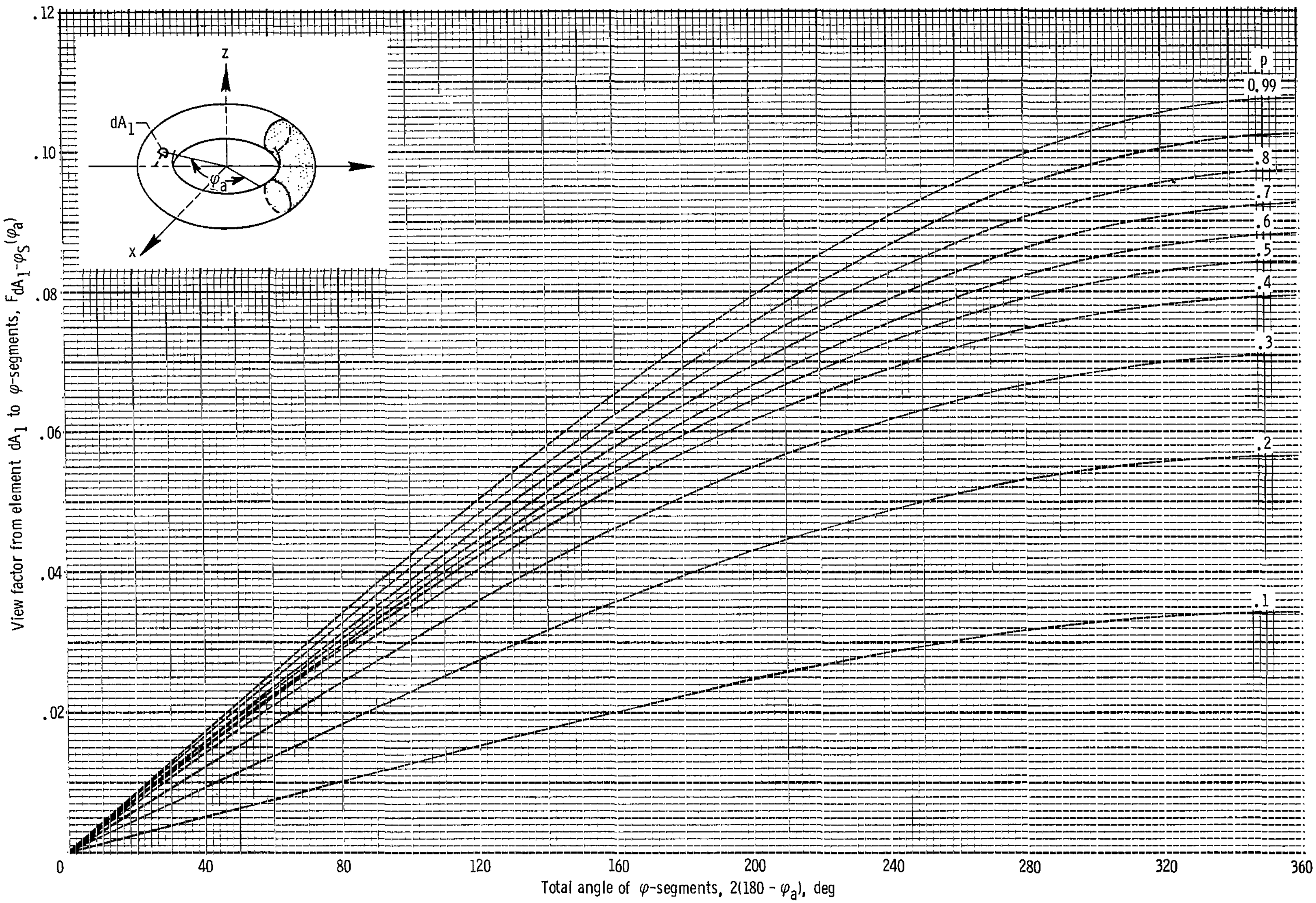
(e) $\alpha_1 = 40°$.

Figure 2. - Continued.



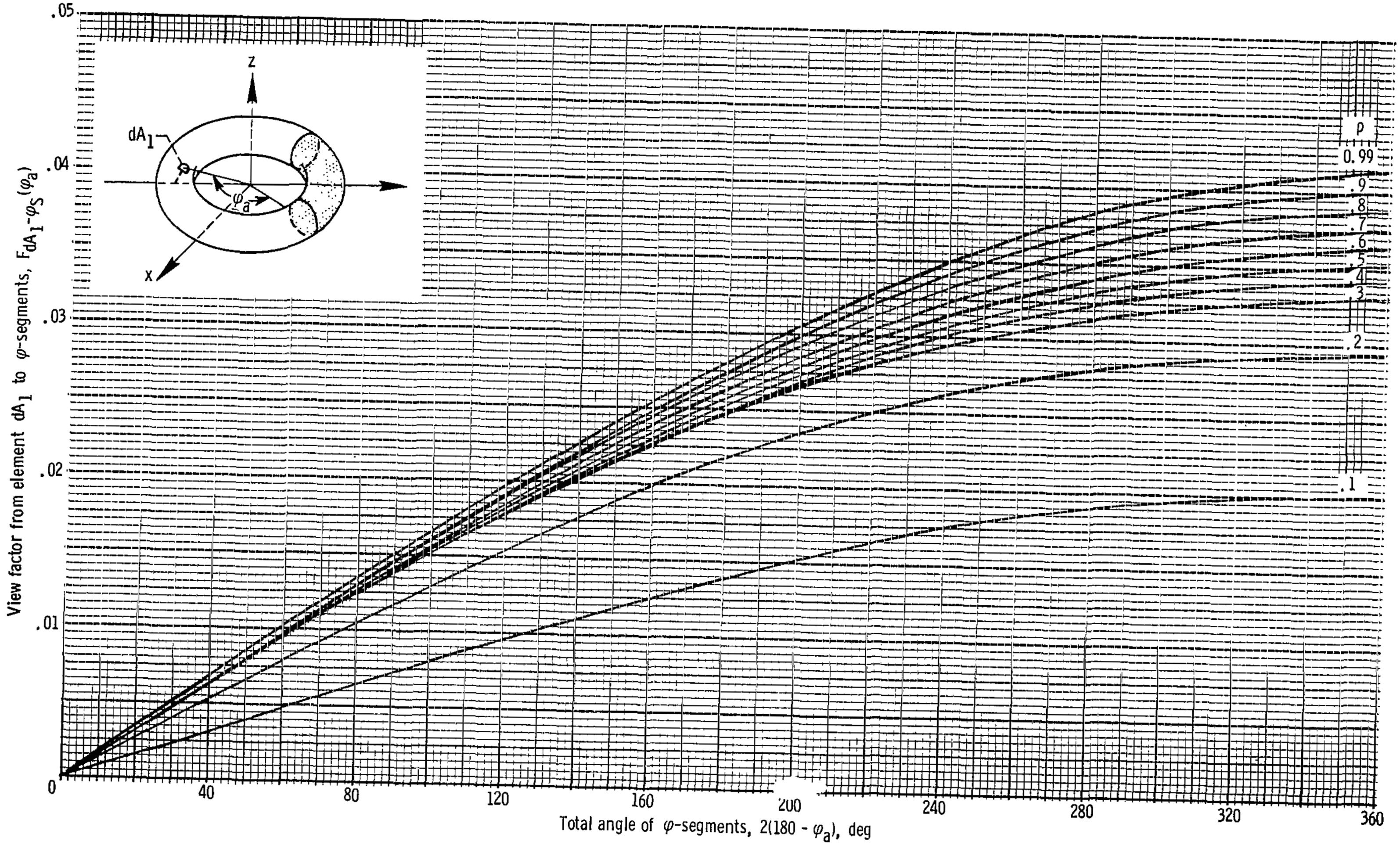
(f) $\alpha_1 = 50°$.

Figure 2. - Continued.



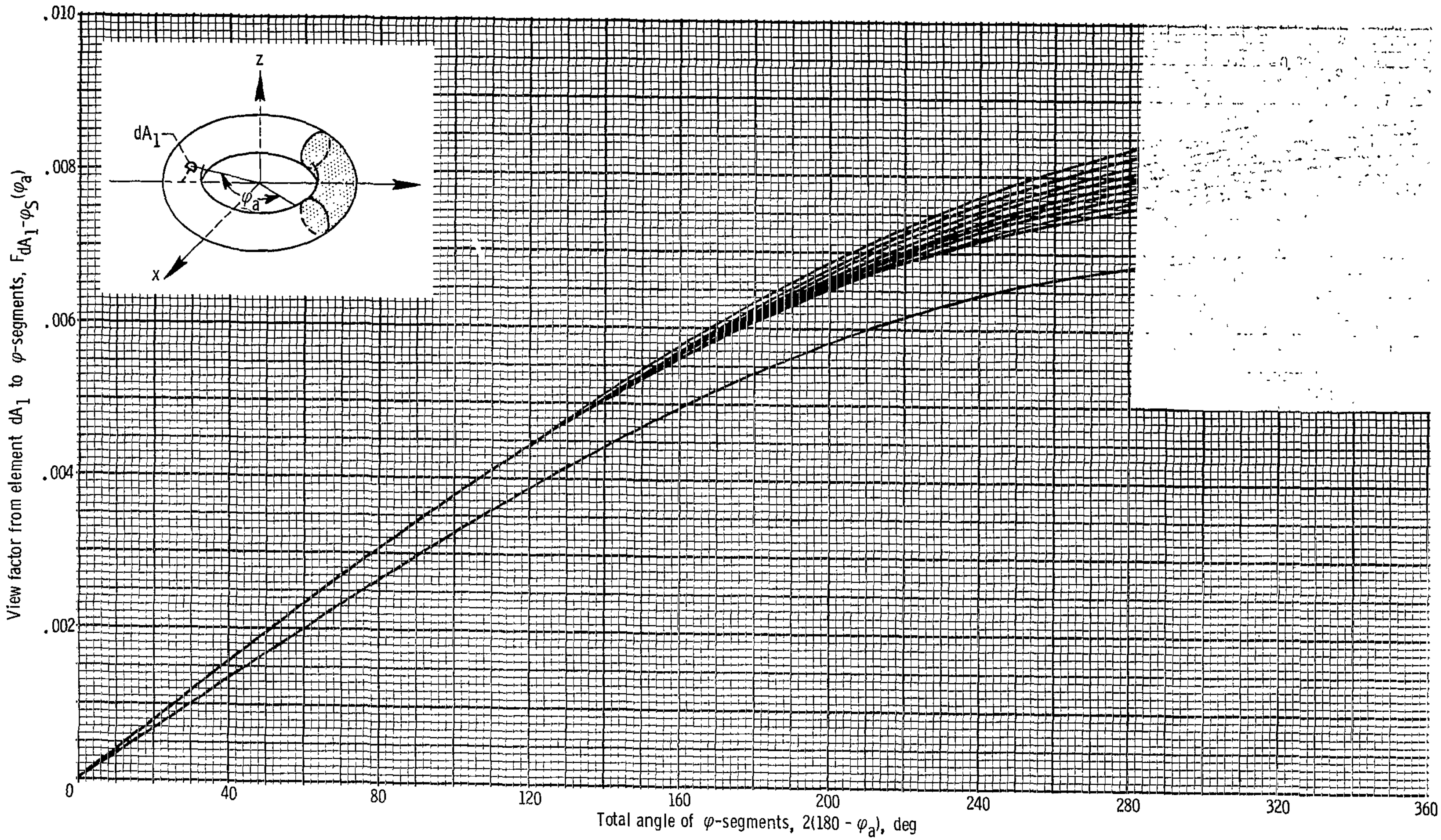
(g) $\alpha_1 = 60°$.

Figure 2. - Continued.



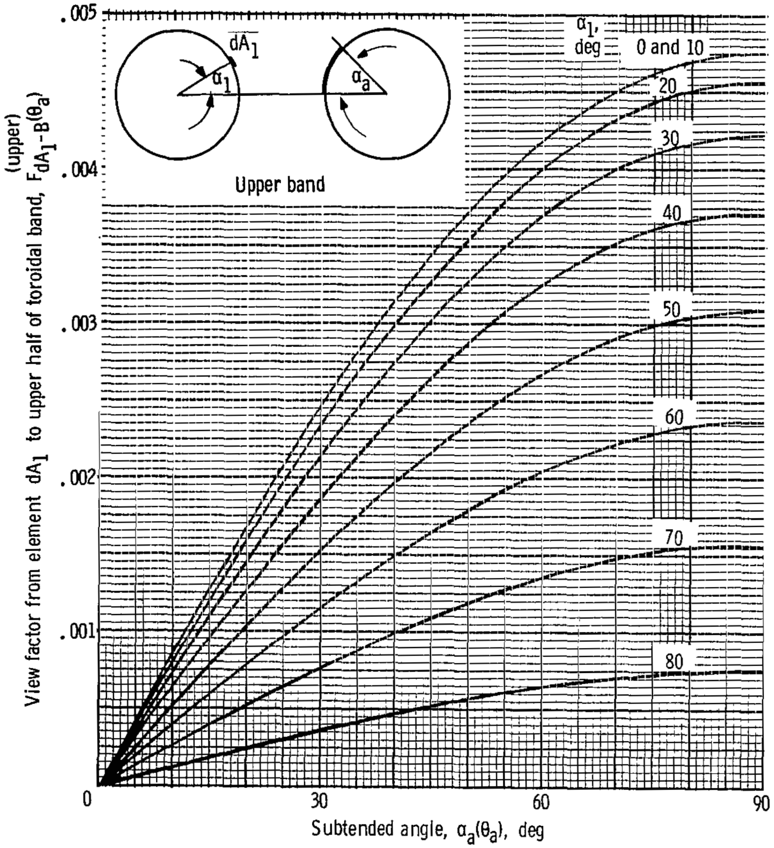
(h) $\alpha_1 = 70°$.

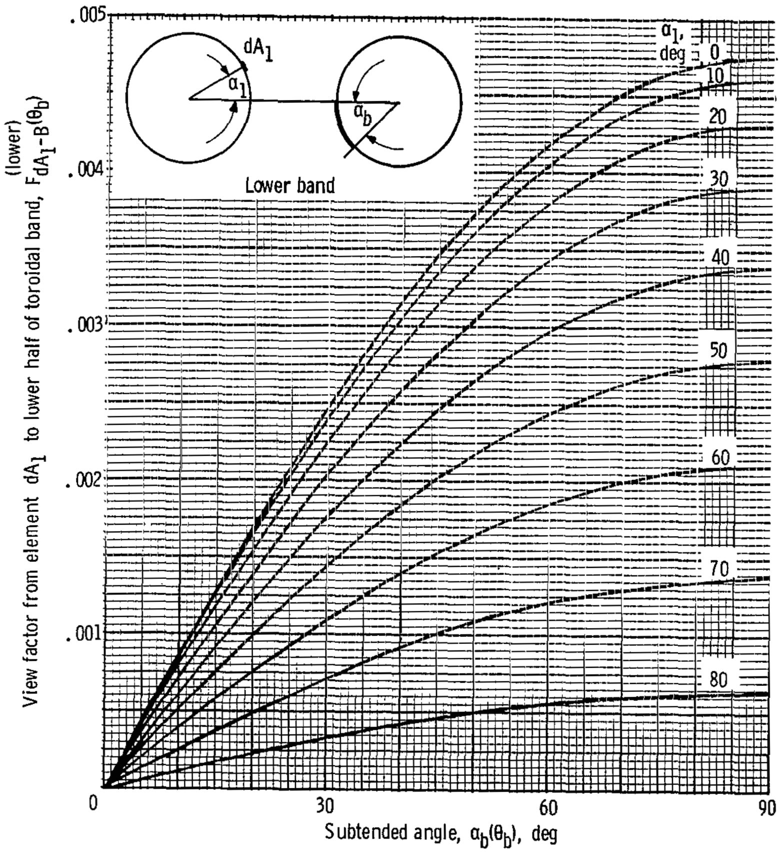
Figure 2. - Continued.



(i) $\alpha_1 = 70^\circ$.

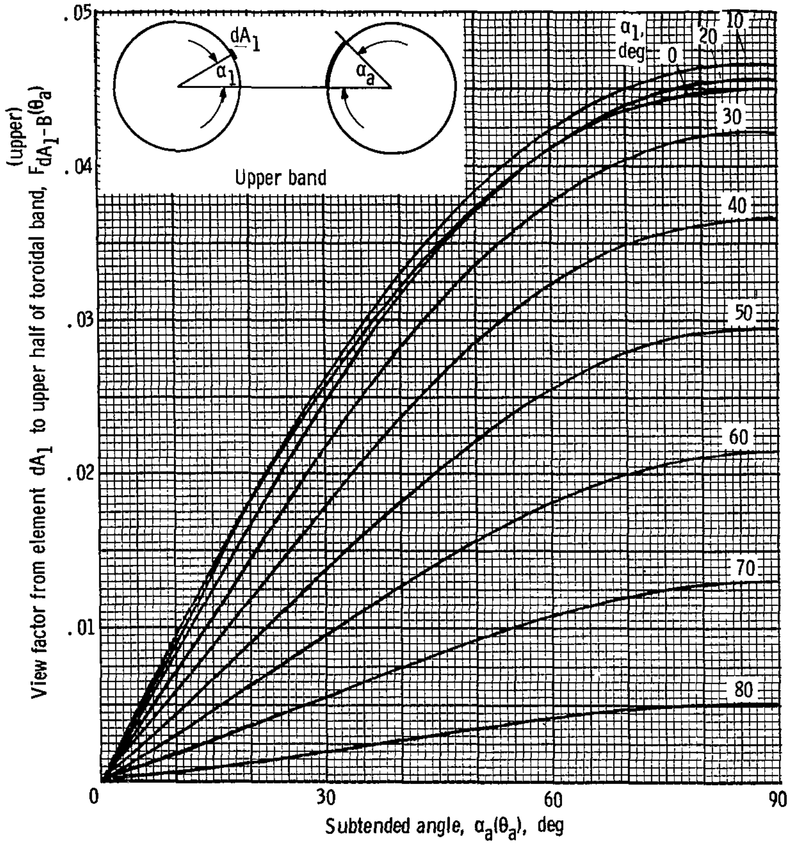
Figure 2. - Concluded.

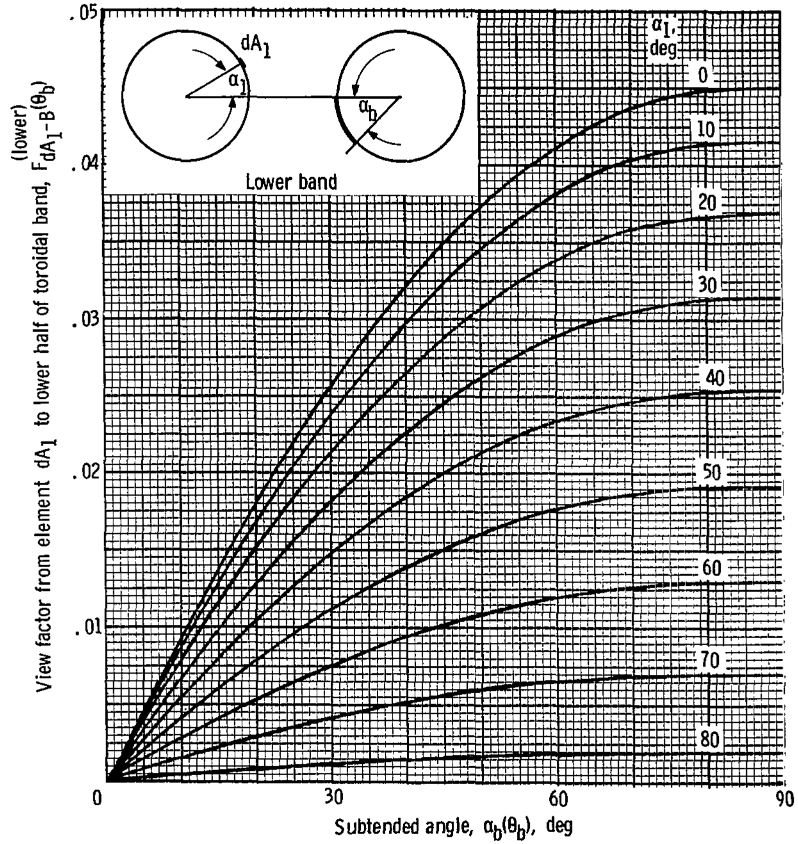




(a) $\rho = 0.01$

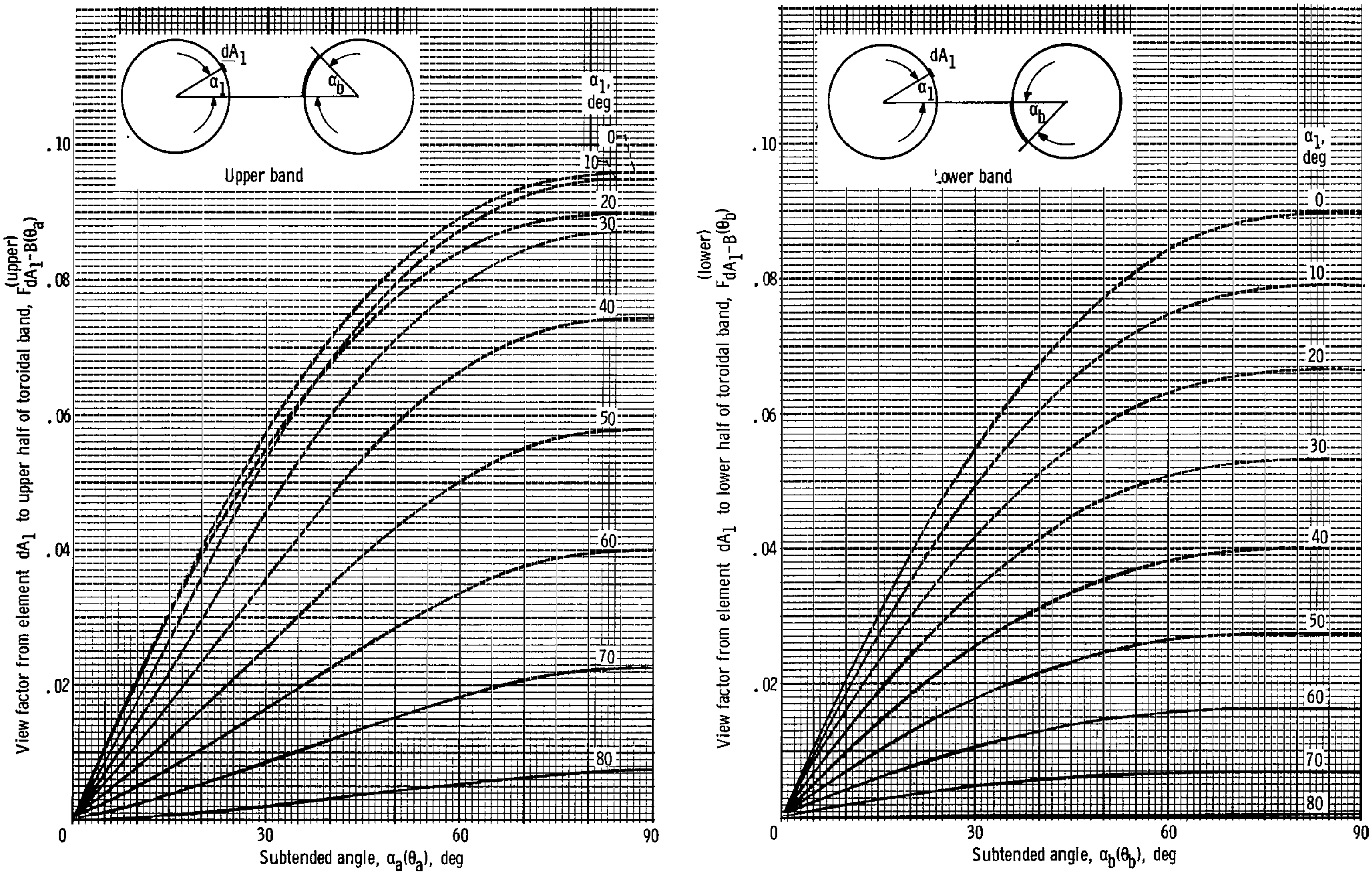
Figure 3. - View factor from element dA_1 to upper and lower half of toroidal band as function of angle α subtended by each half of the band with α_1 as a parameter.





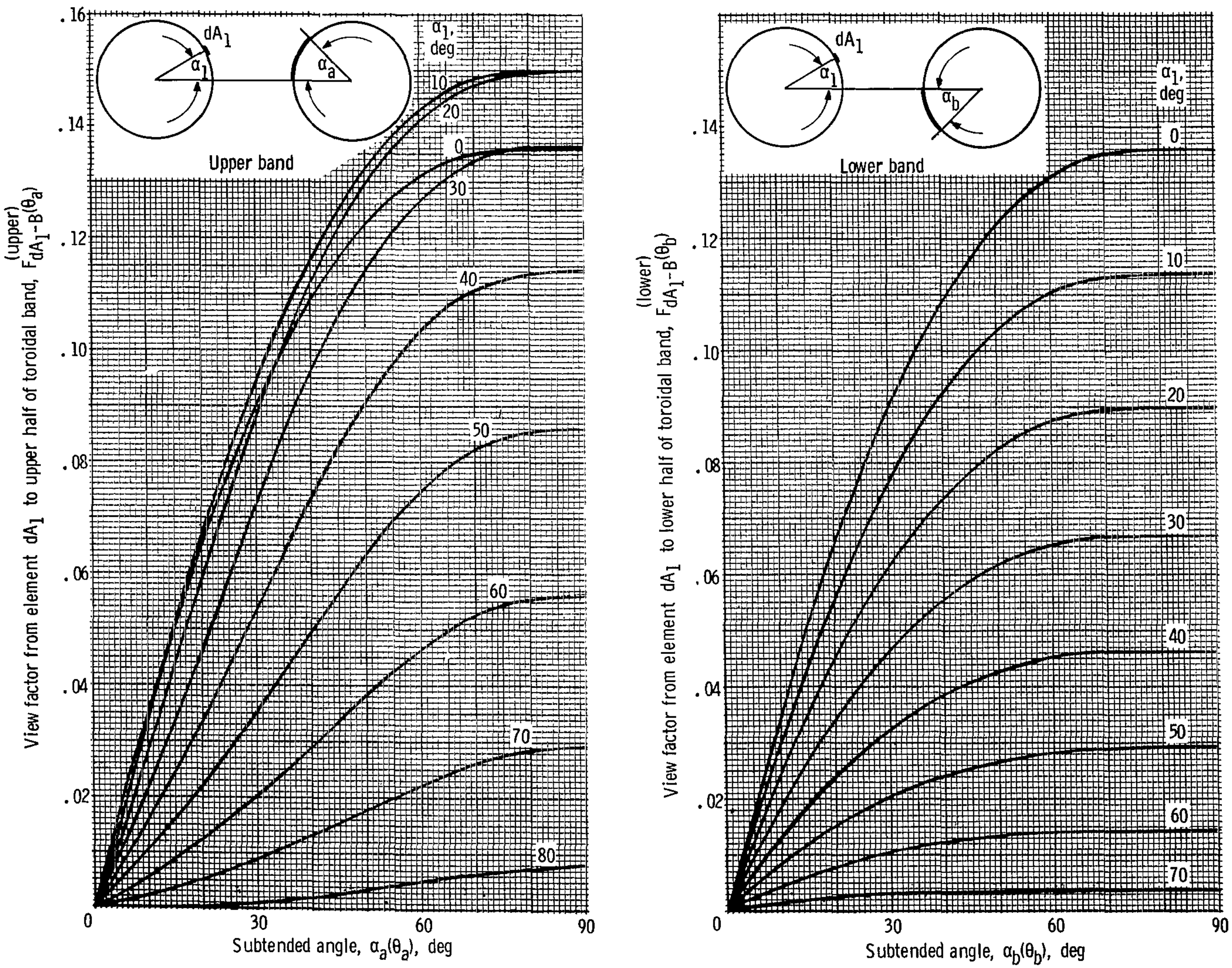
(b) $\rho = 0.1$.

Figure 3. - Continued.



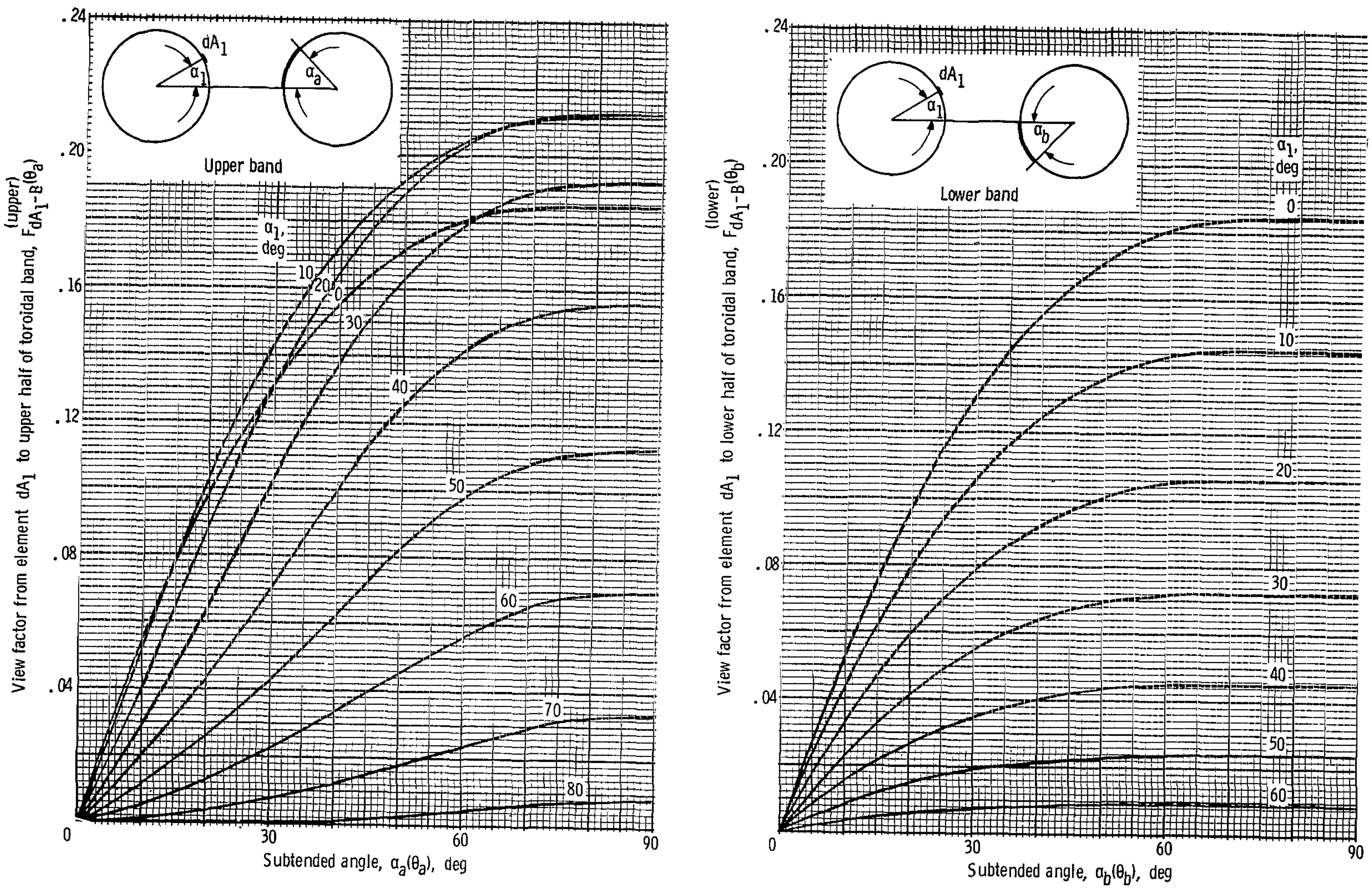
(c) $\rho = 0.2$.

Figure 3. - Continued.



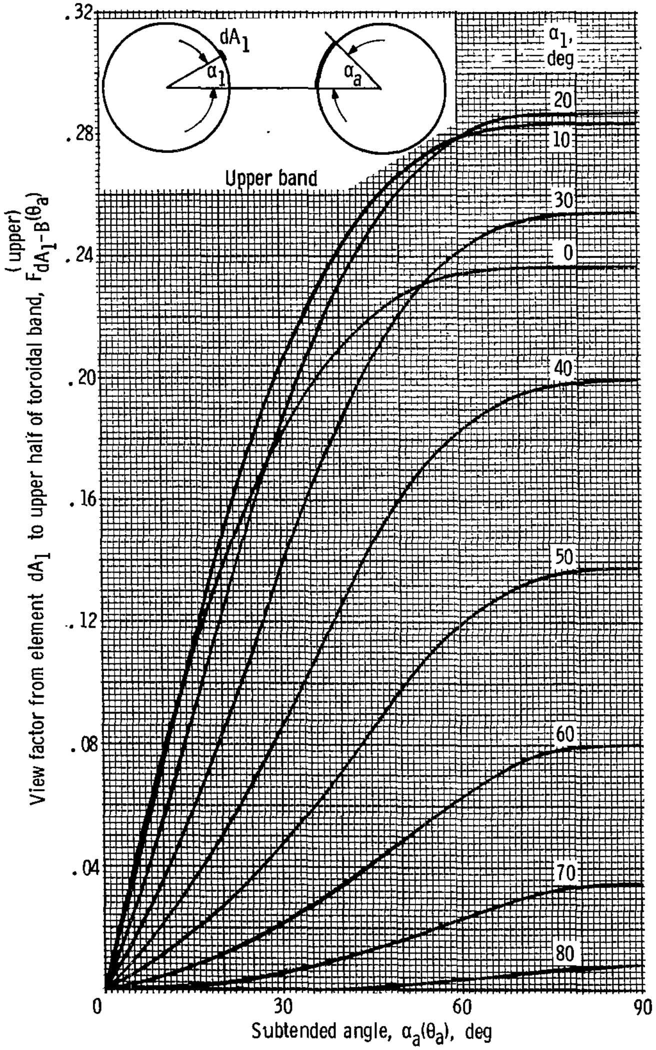
(d) $\rho = 0.3$.

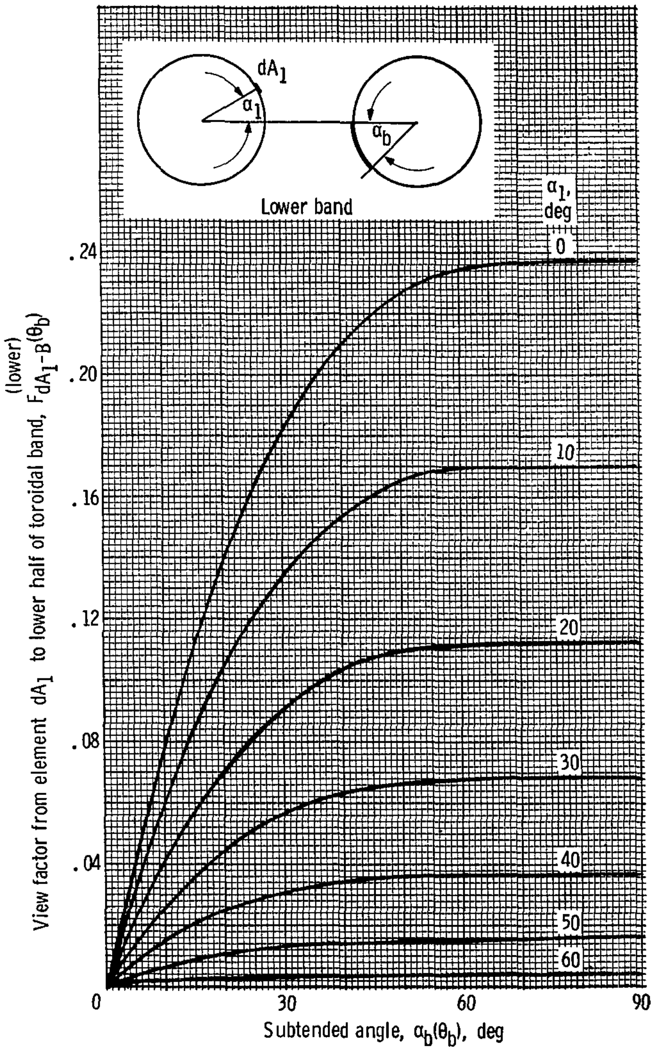
Figure 3. - Continued.



(e) $\rho = 0.4$.

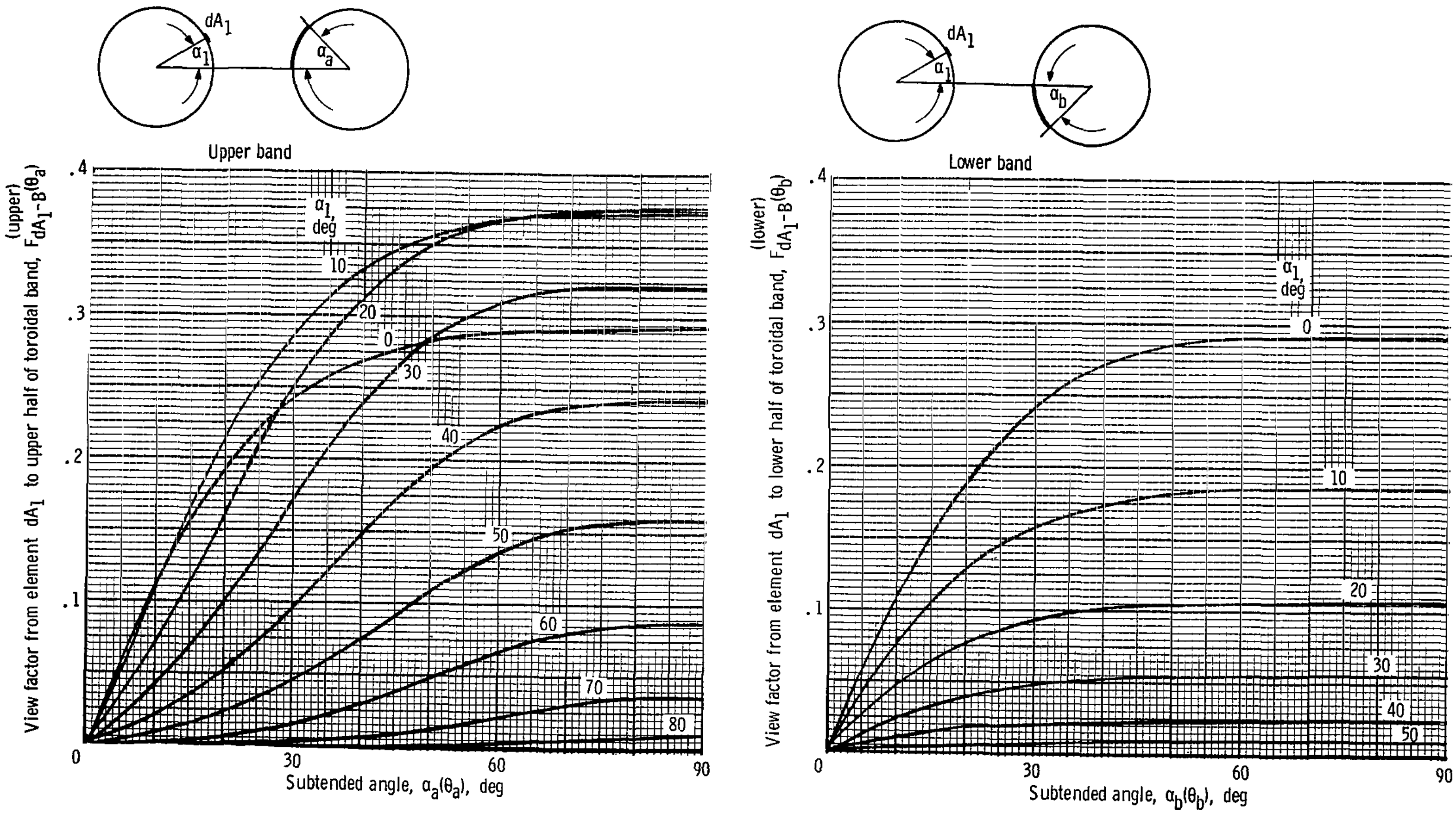
Figure 3. - Continued.





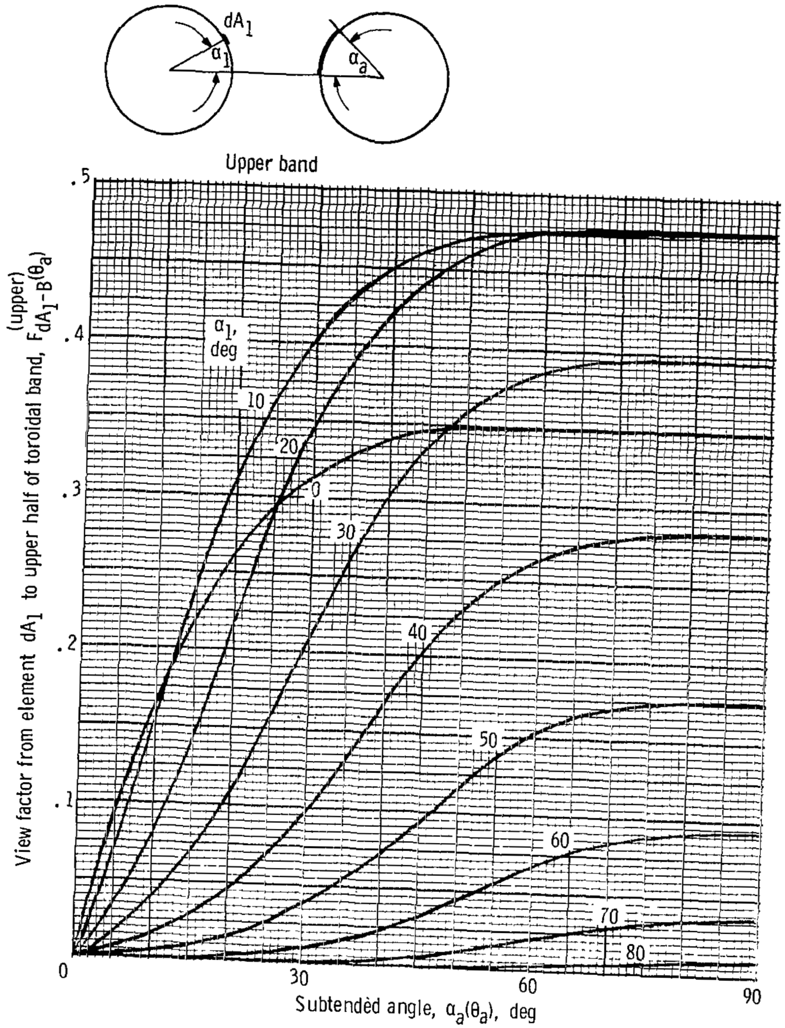
(f) $\rho = 0.5$.

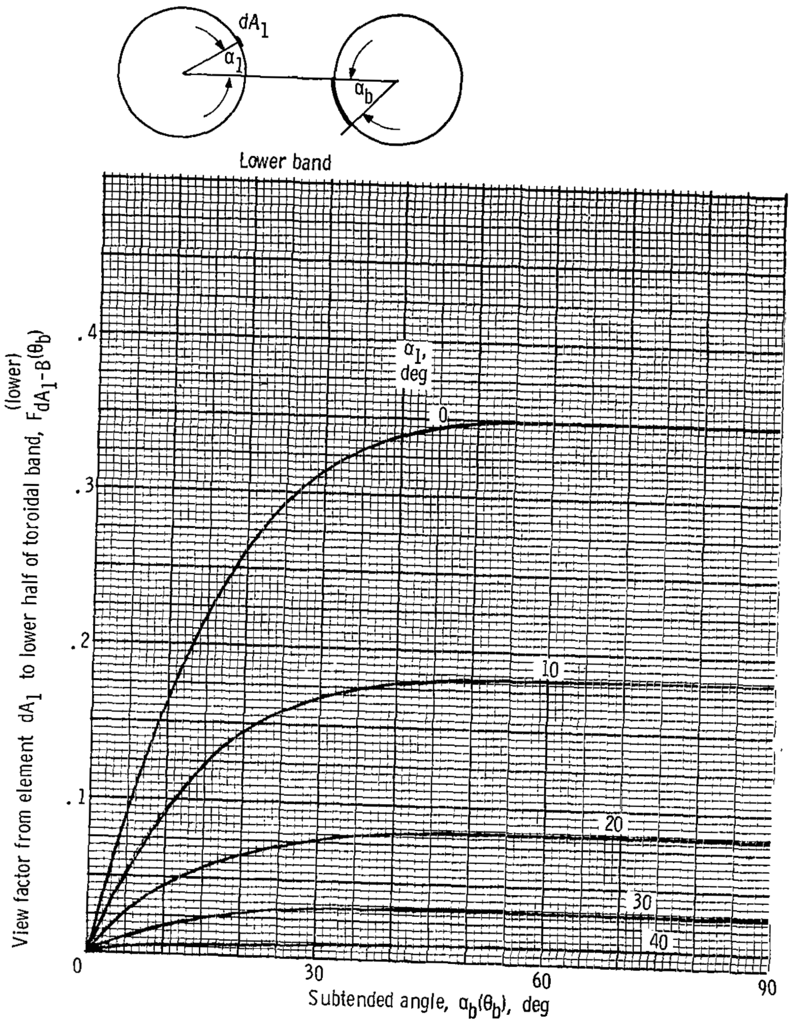
Figure 3. - Continued.



(g) $\rho = 0.6$.

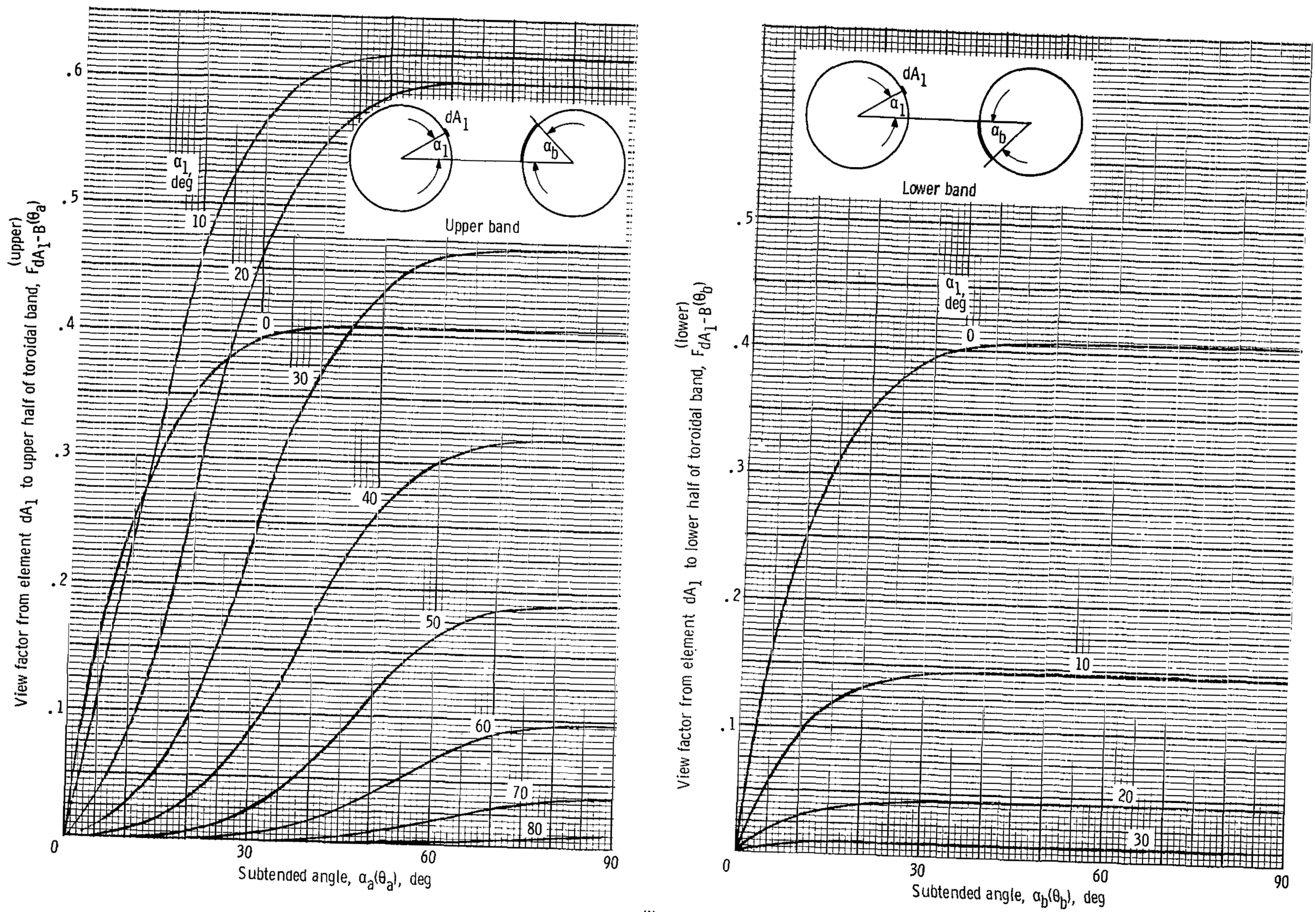
Figure 3. - Continued.





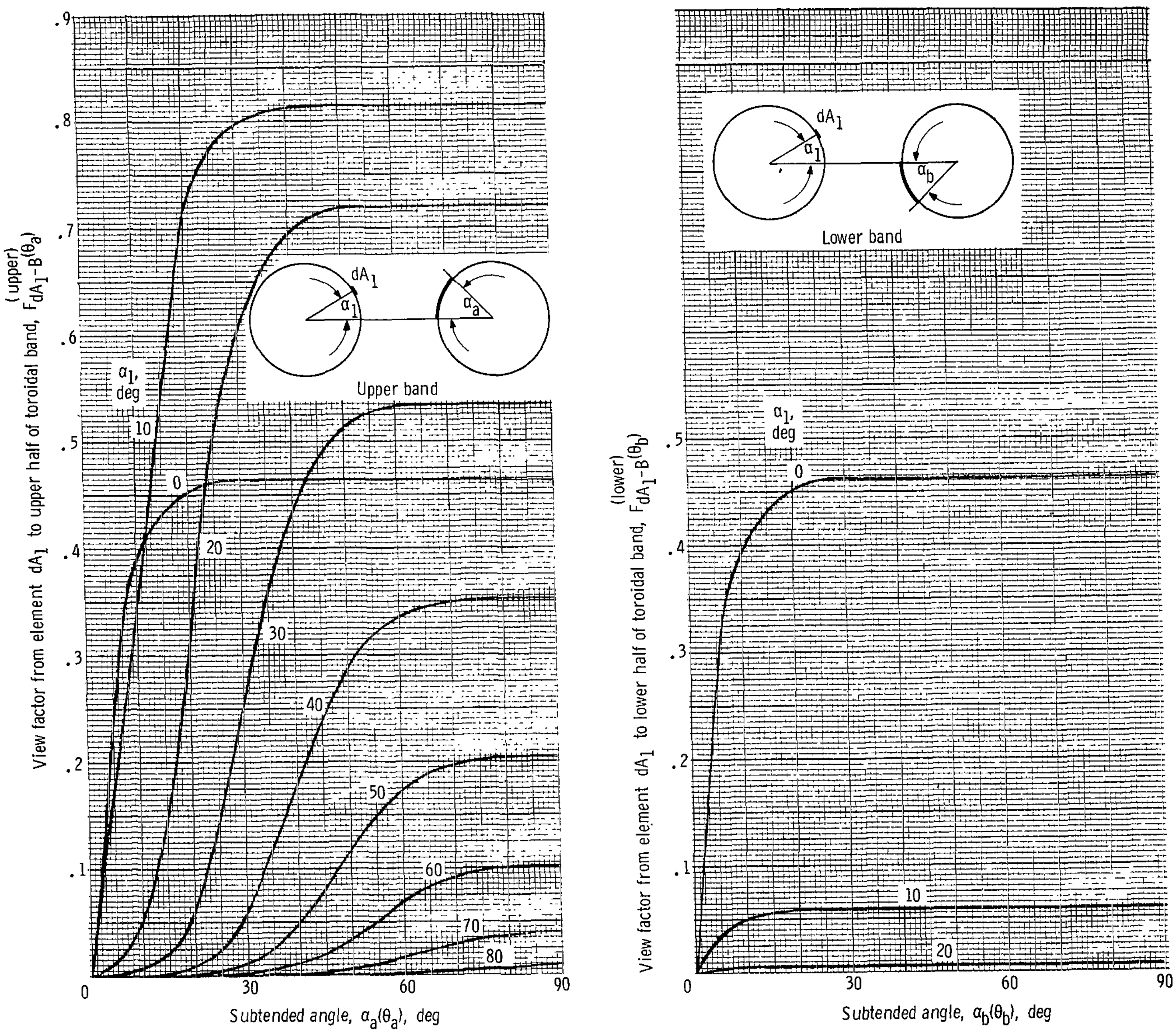
(h) $\rho = 0.7$.

Figure 3. - Continued.



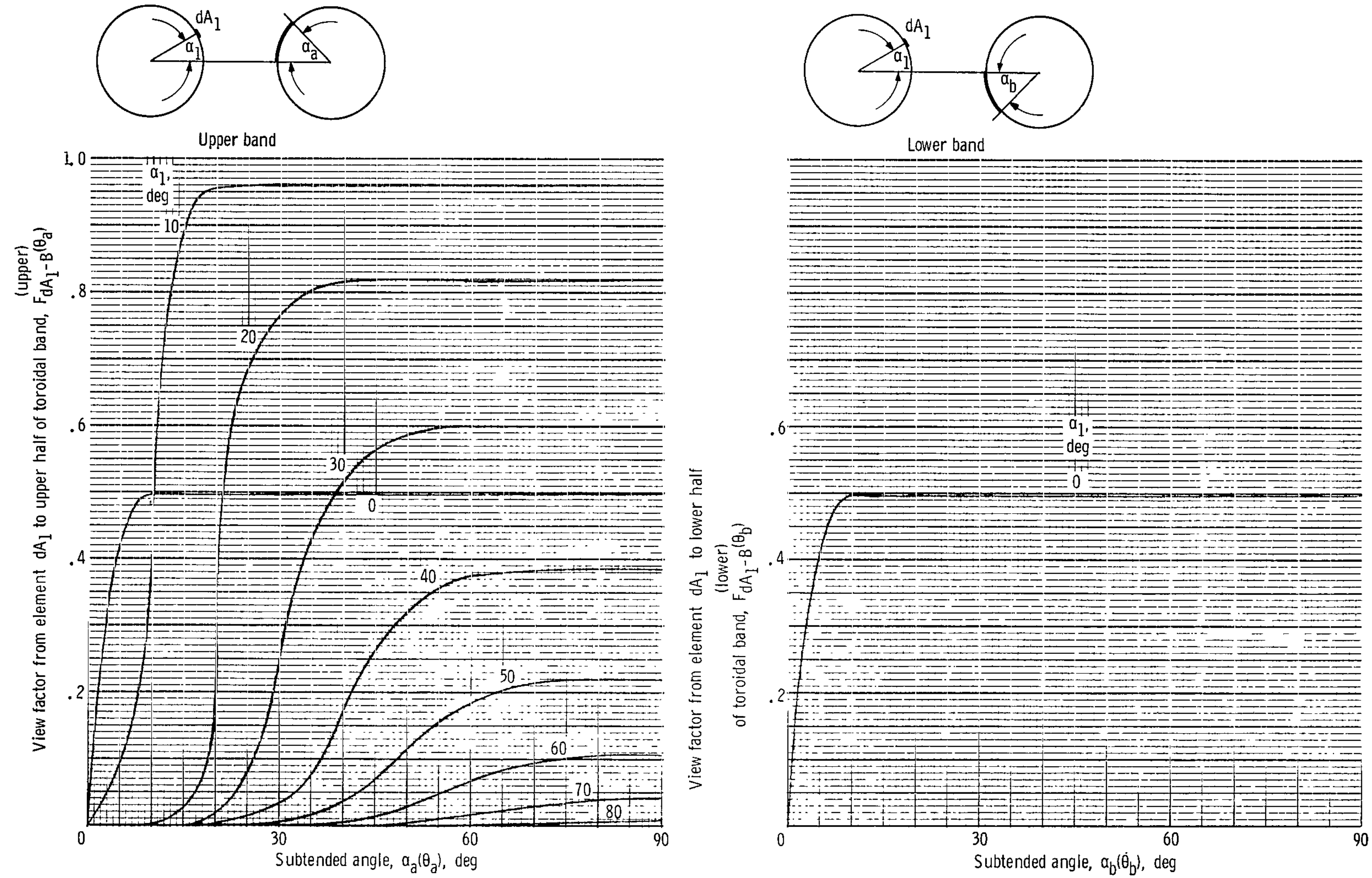
(i) $p = 0.8$.

Figure 3. - Continued.



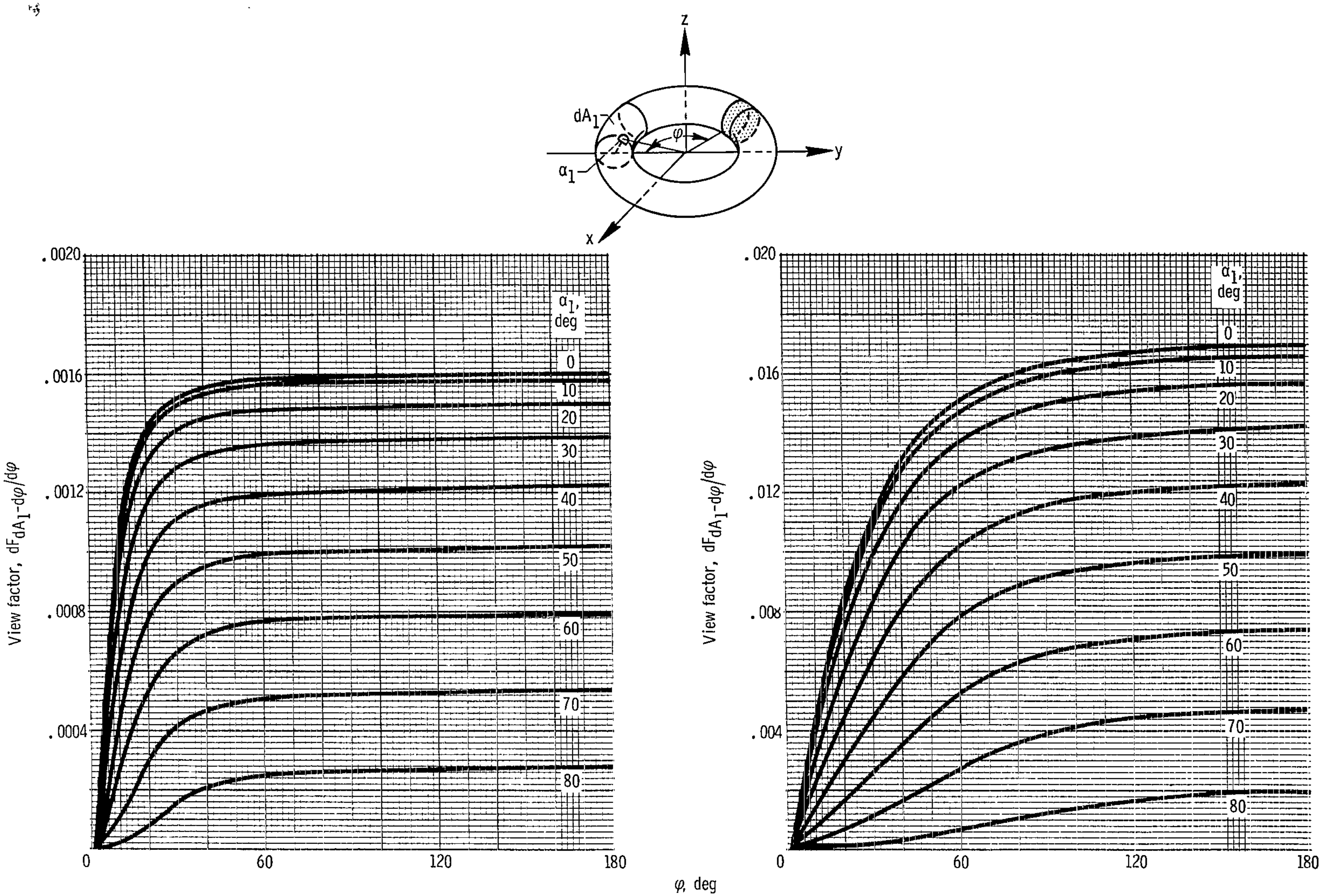
(j) $\rho = 0.9$

Figure 3. - Continued.



(k) $\rho = 0.99$.

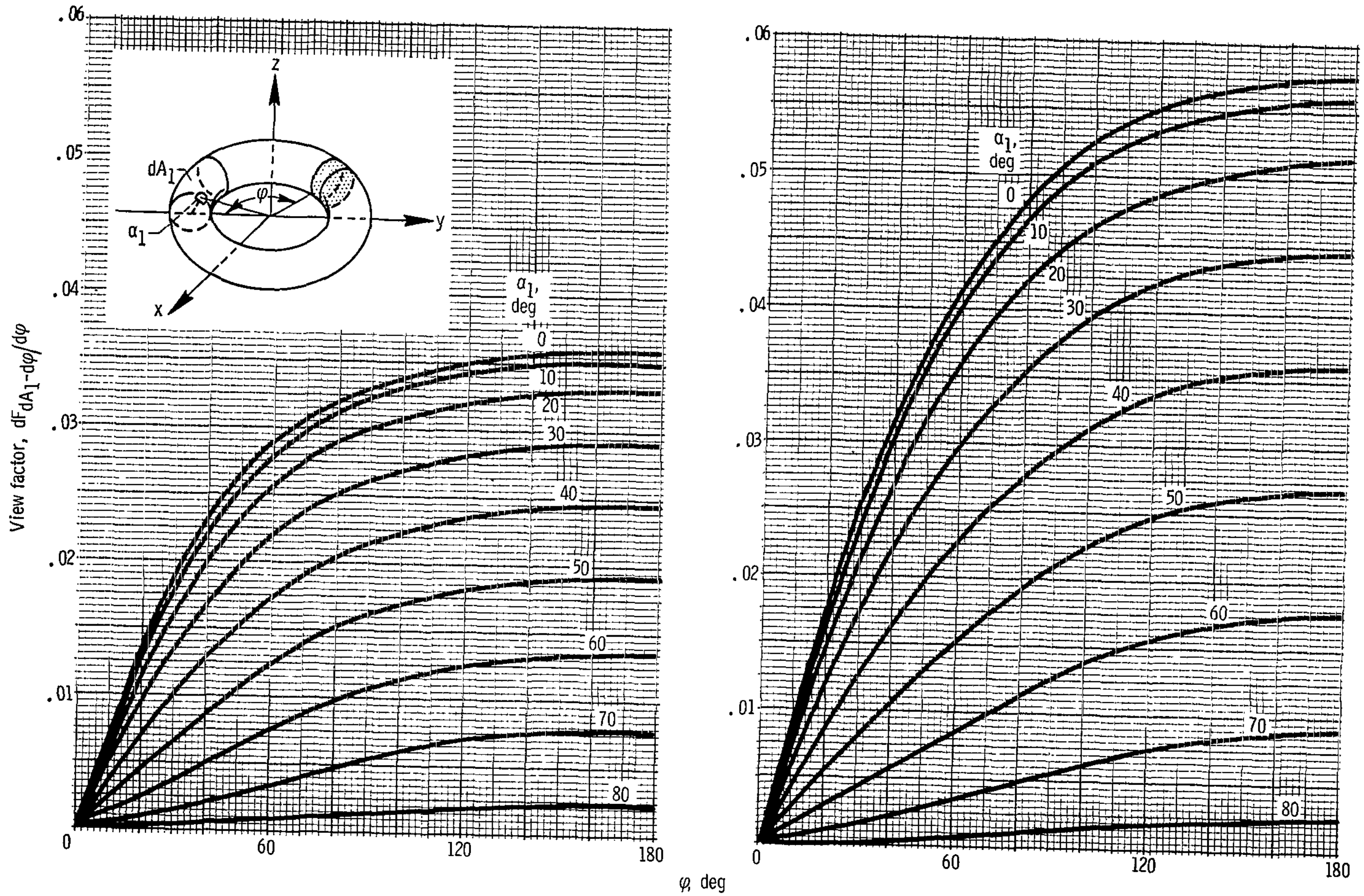
Figure 3. - Concluded.



(a) $\rho = 0.01$.

(b) $\rho = 0.10$.

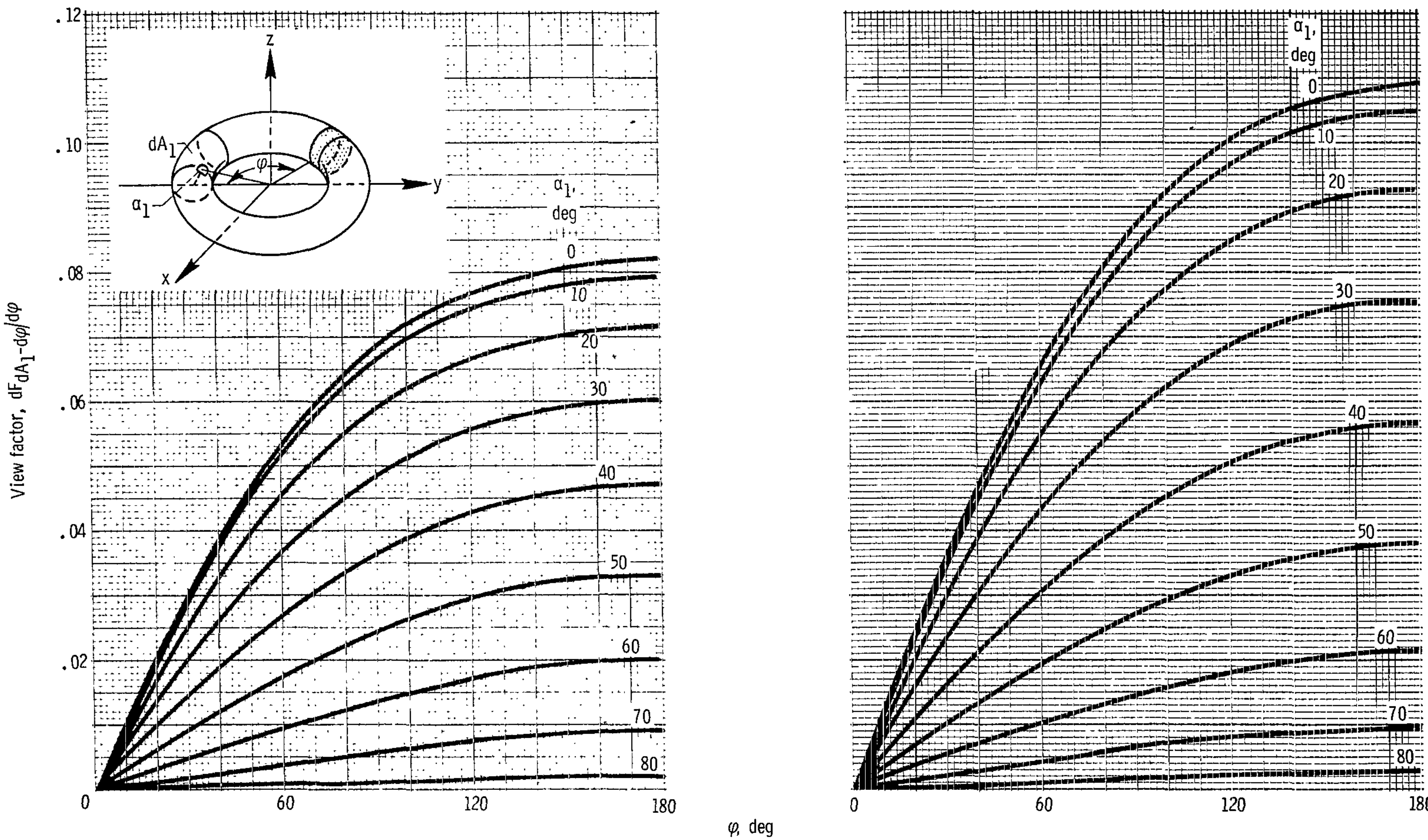
Figure 4. - View factor from element dA_1 to differential φ-segment.



(c) $\rho = 0.20$.

(d) $\rho = 0.30$.

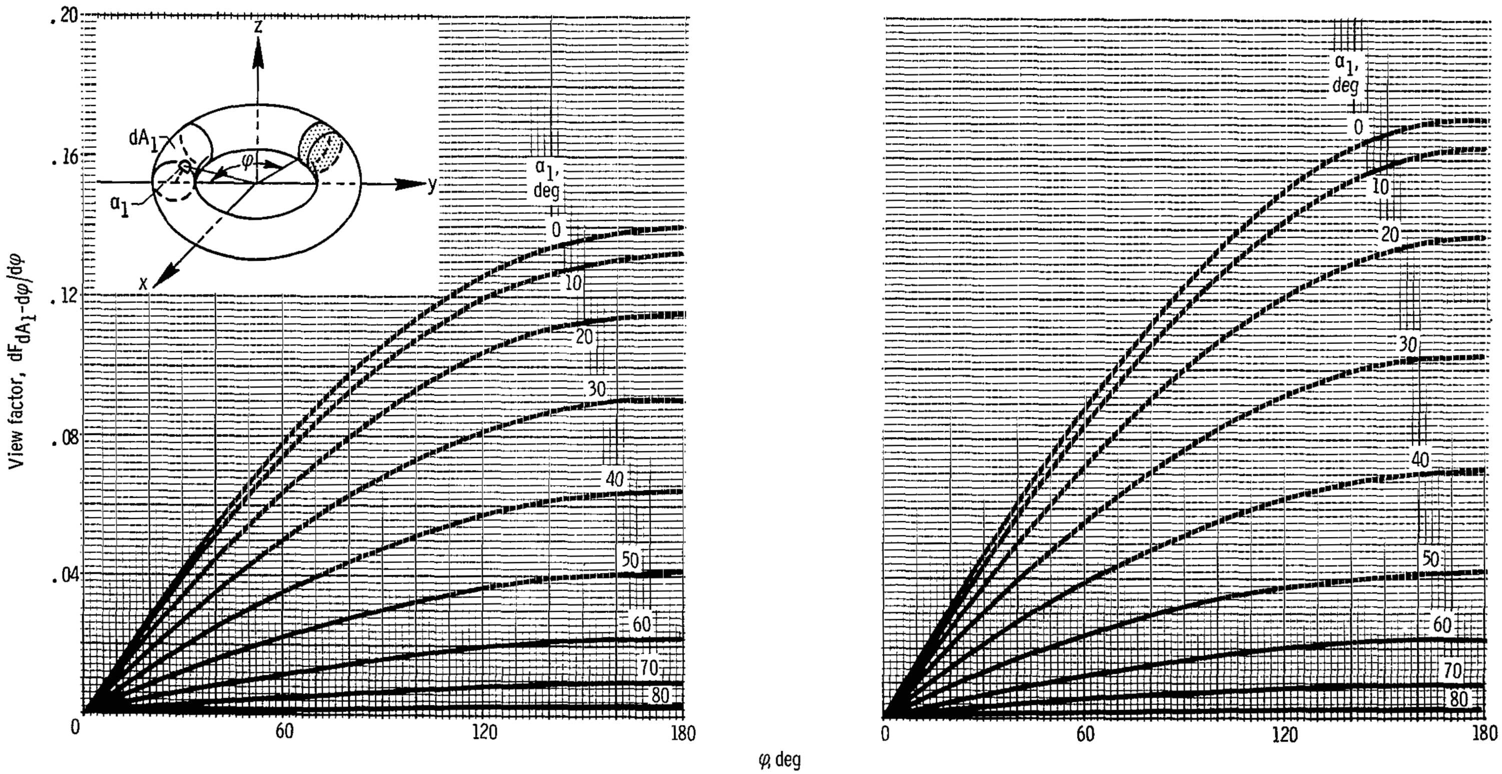
Figure 4. - Continued.



(e) ρ = 0.40.

(f) ρ = 0.50.

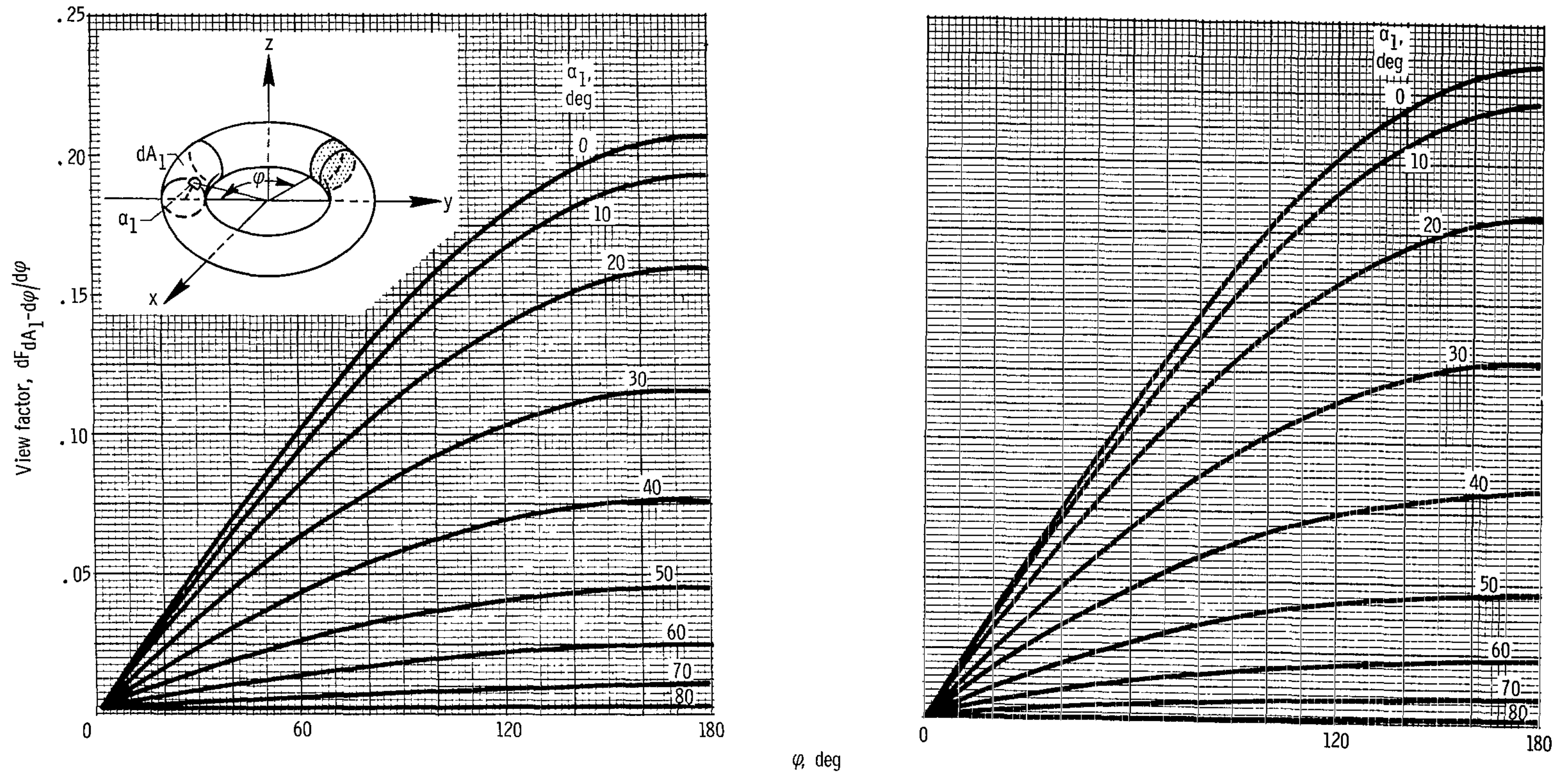
Figure 4. - Continued.



(g) $\rho = 0.60$.

(h) $\rho = 0.70$.

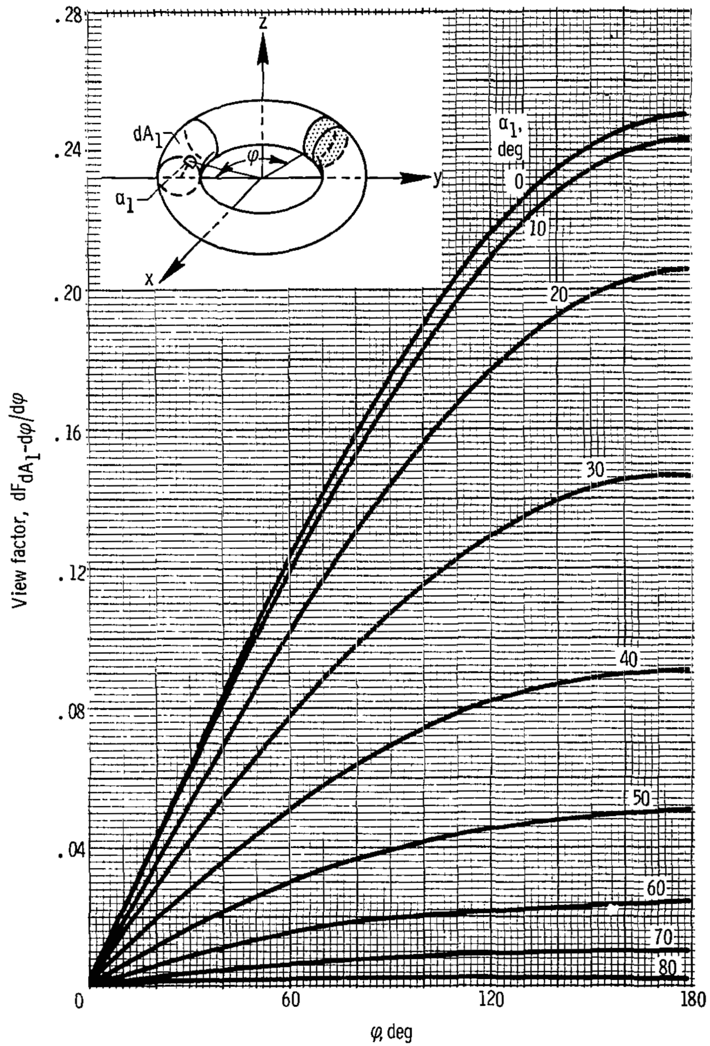
Figure 4. - Continued.



(i) $\rho = 0.80$.

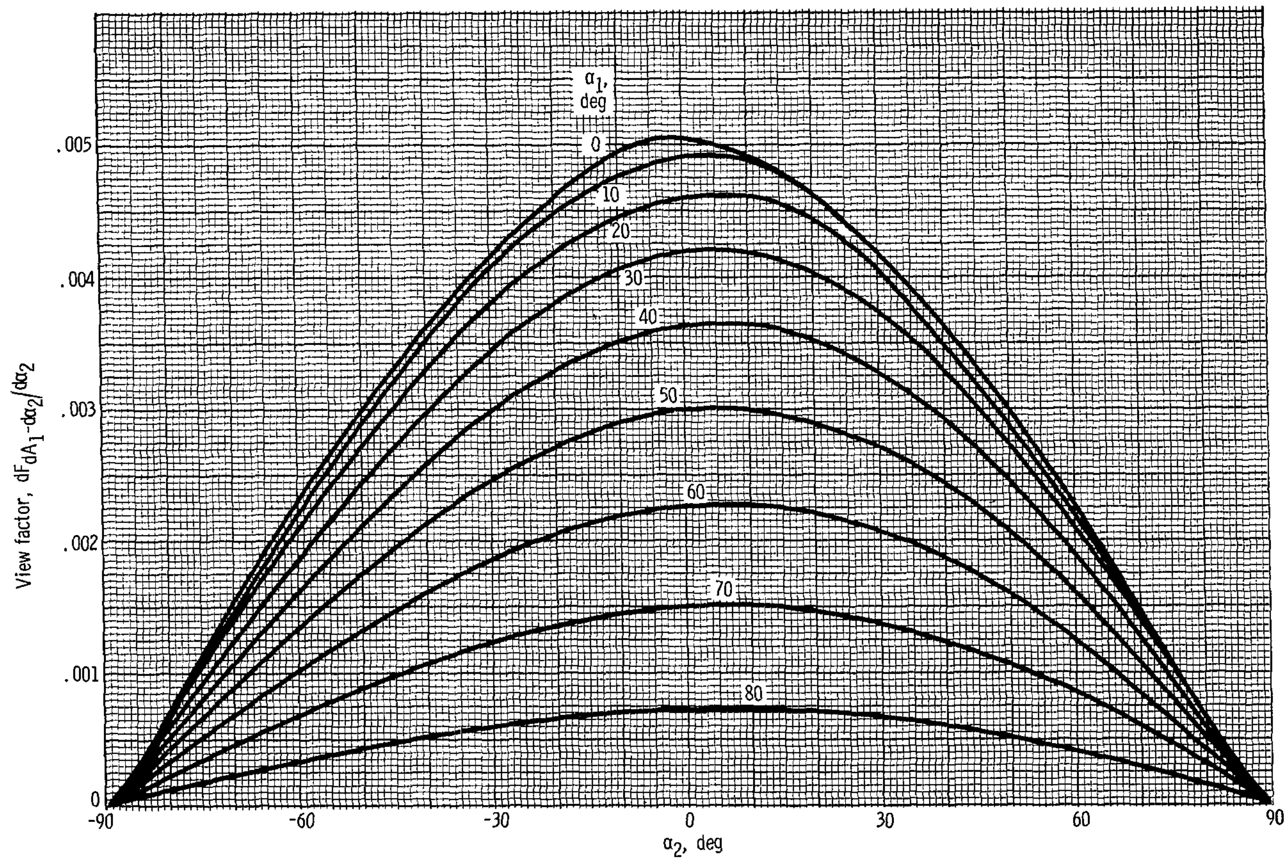
(j) $\rho = 0.90$.

Figure 4. - Continued.



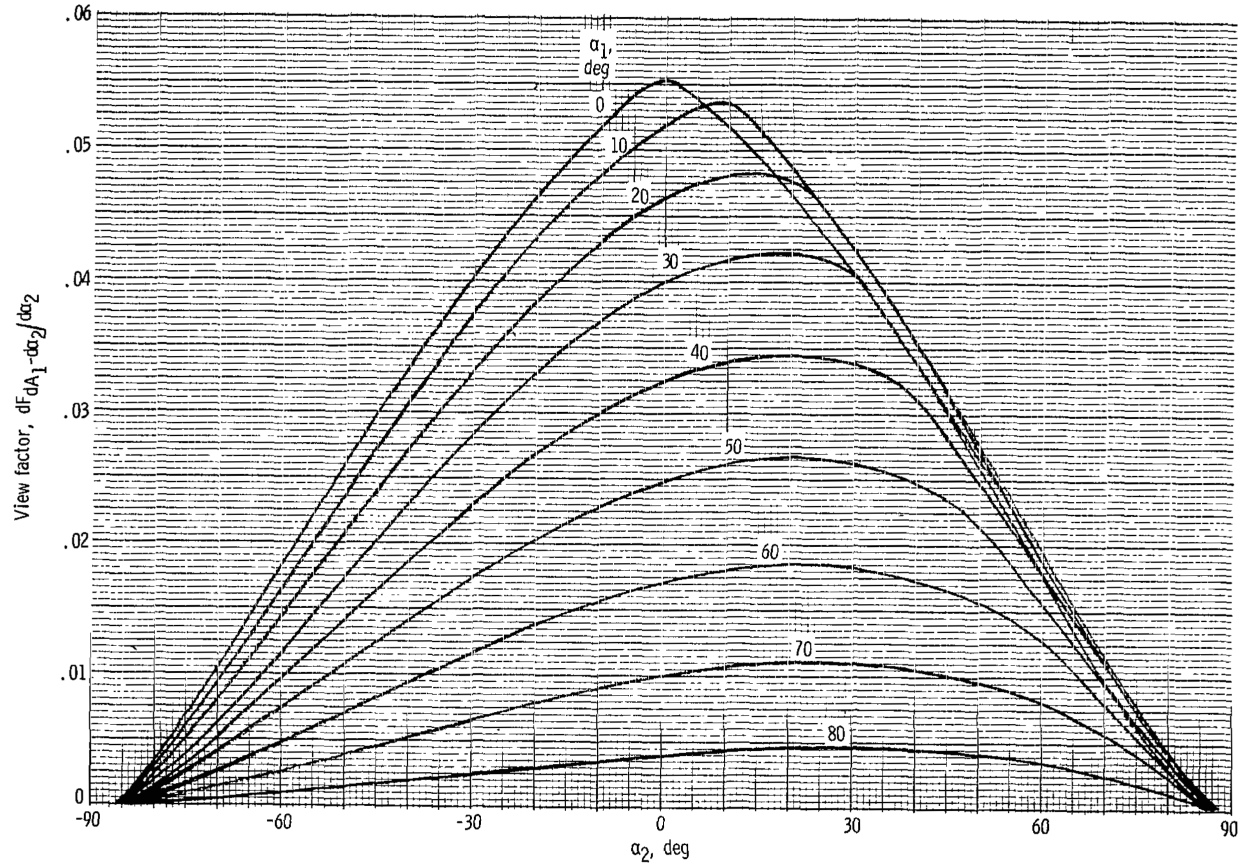
(k) $\rho = 0.99$.

Figure 4. - Concluded.



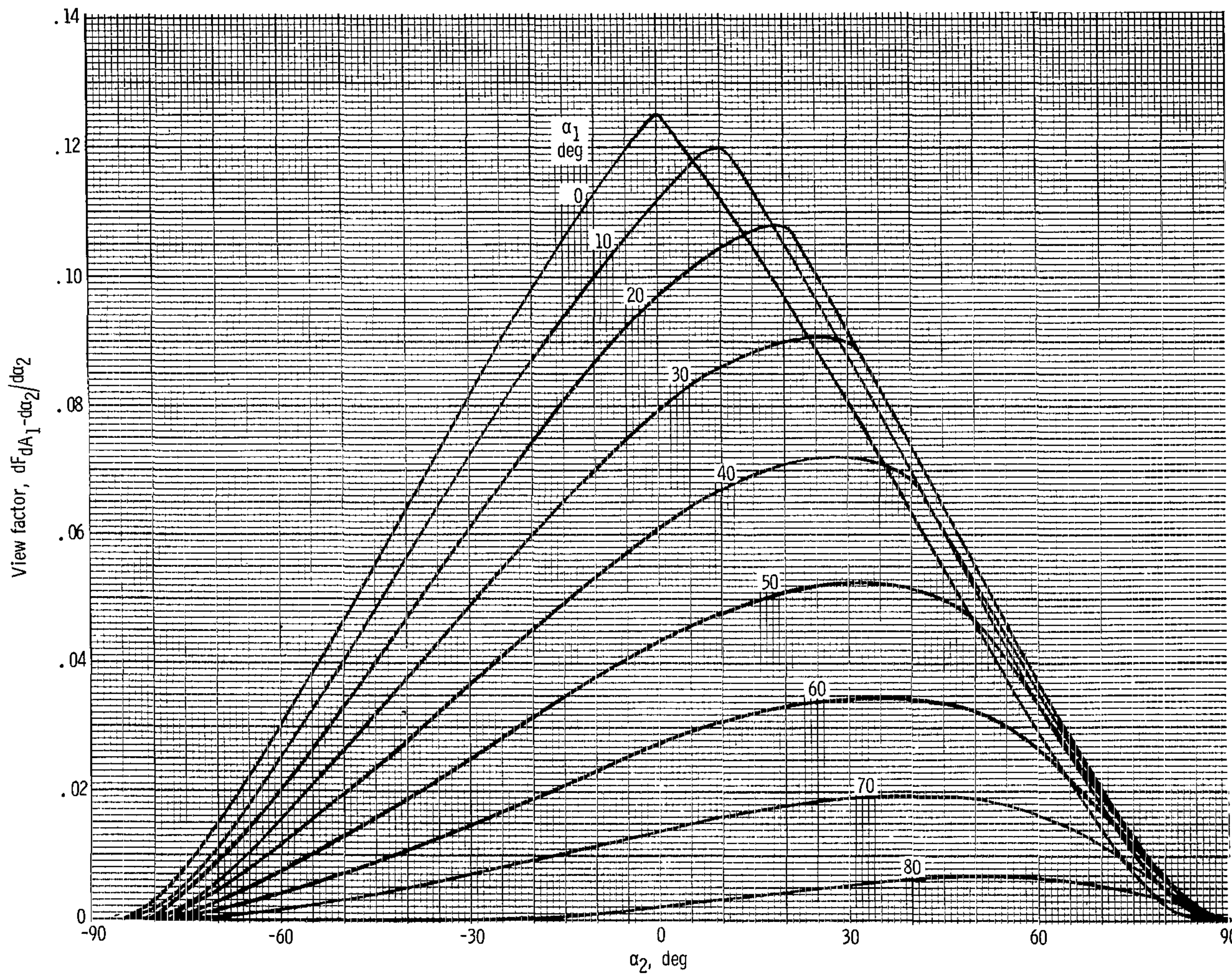
(a) $\rho = 0.01$.

Figure 5. - View factor - element to differential toroidal band.



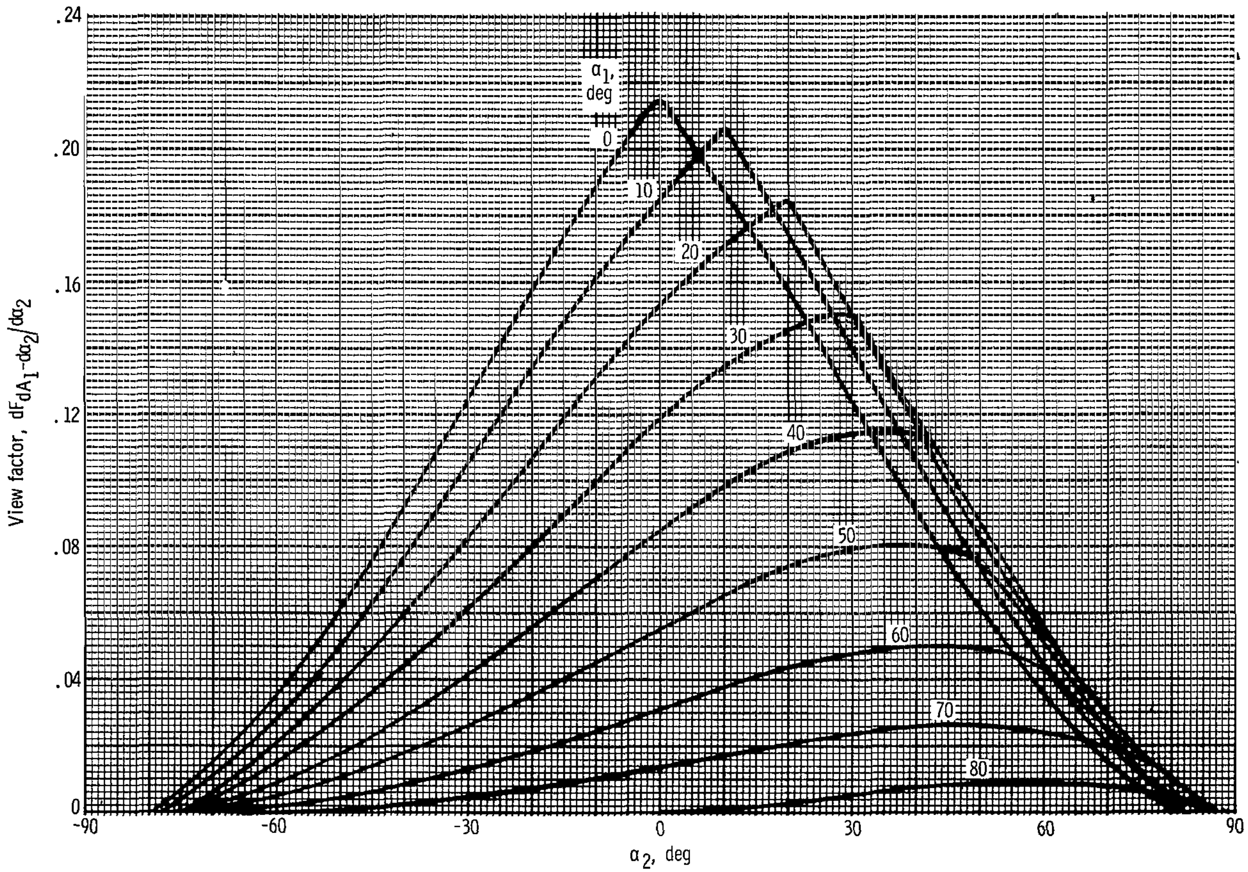
(b) ρ = 0.10.

Figure 5. - Continued.



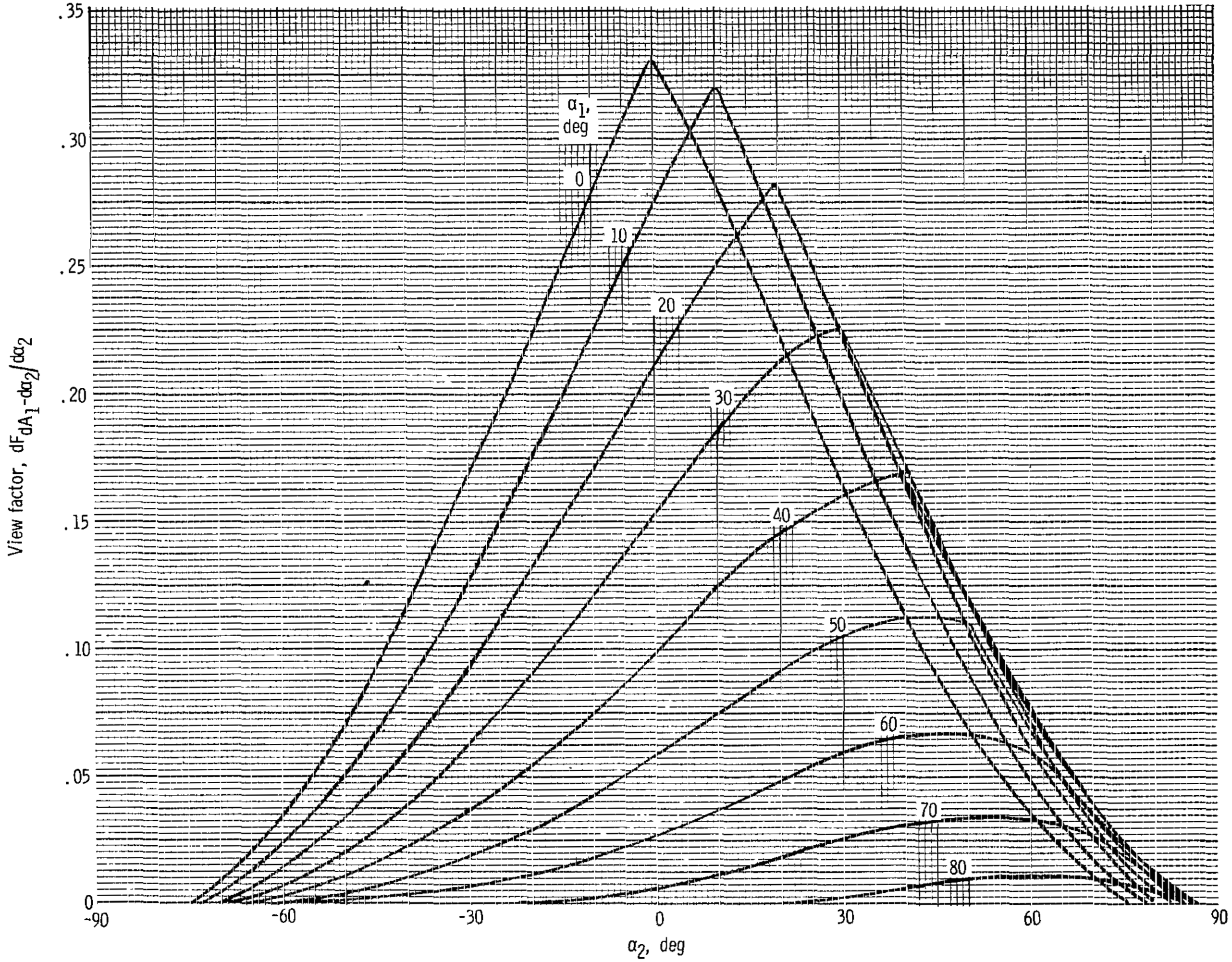
(c) $\rho = 0.20$.

Figure 5. - Continued.



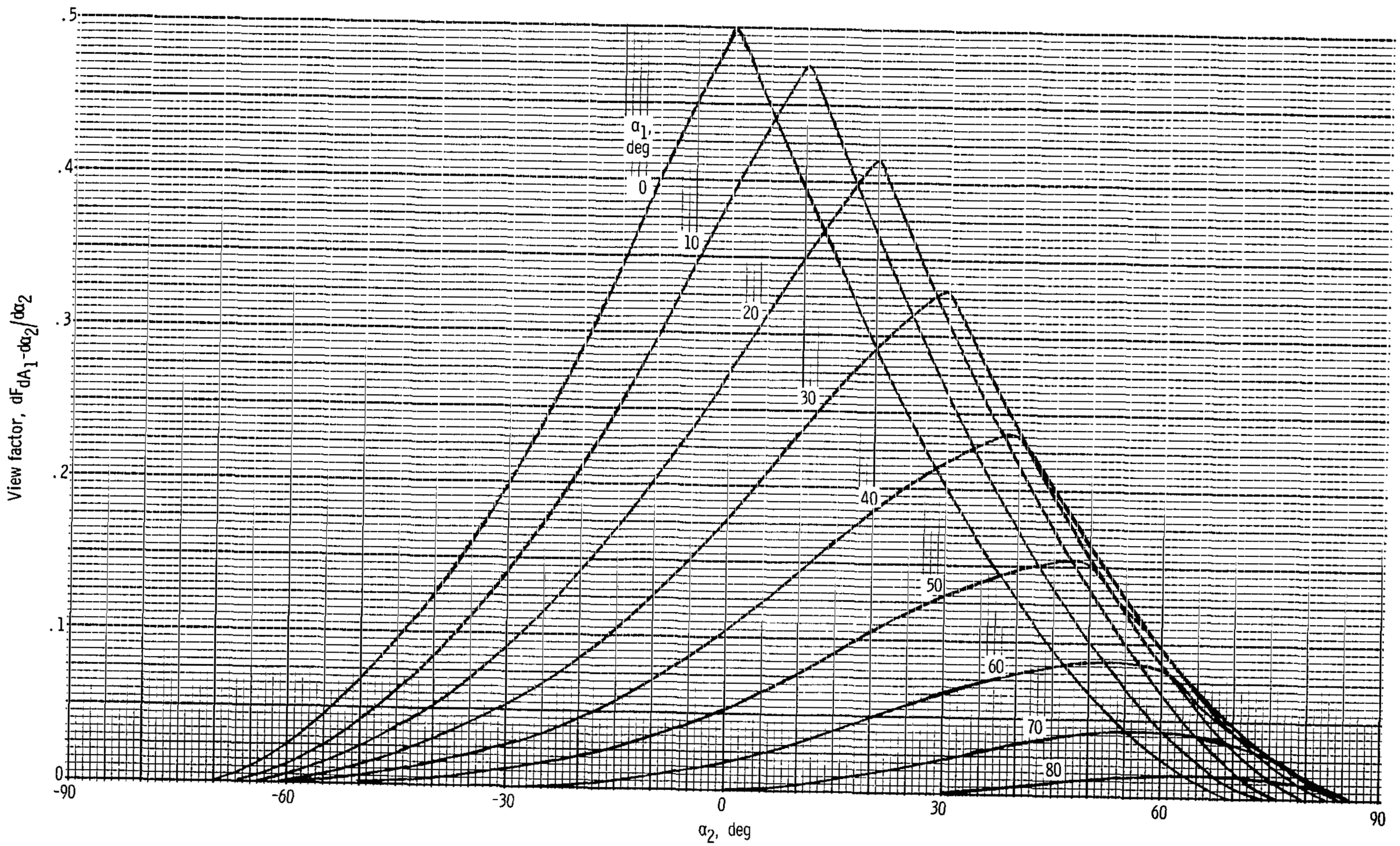
(d) ρ = 0.30.

Figure 5. - Continued.



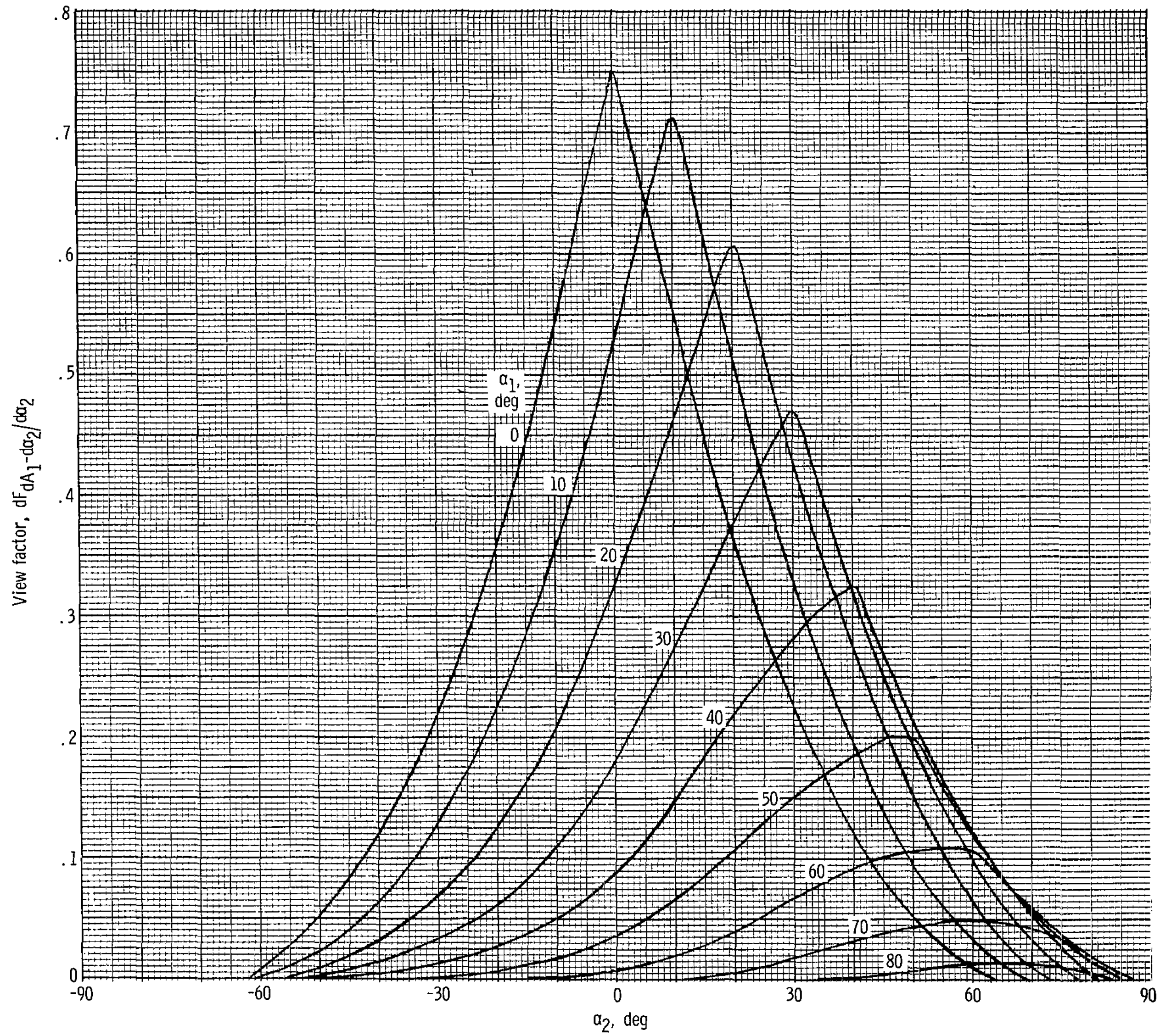
(e) $\rho = 0.40$.

Figure 5. - Continued.



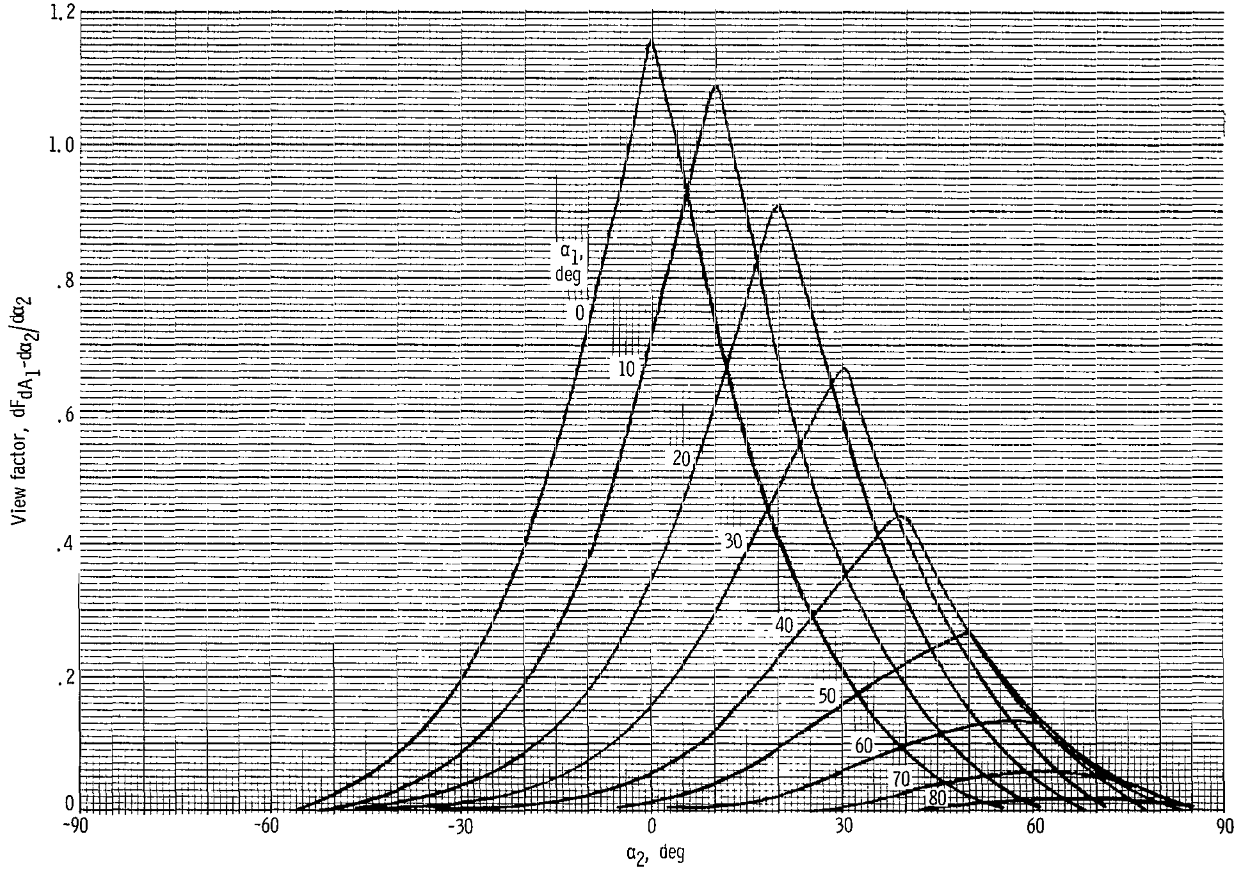
(f) $\rho = 0.50$.

Figure 5. - Continued.



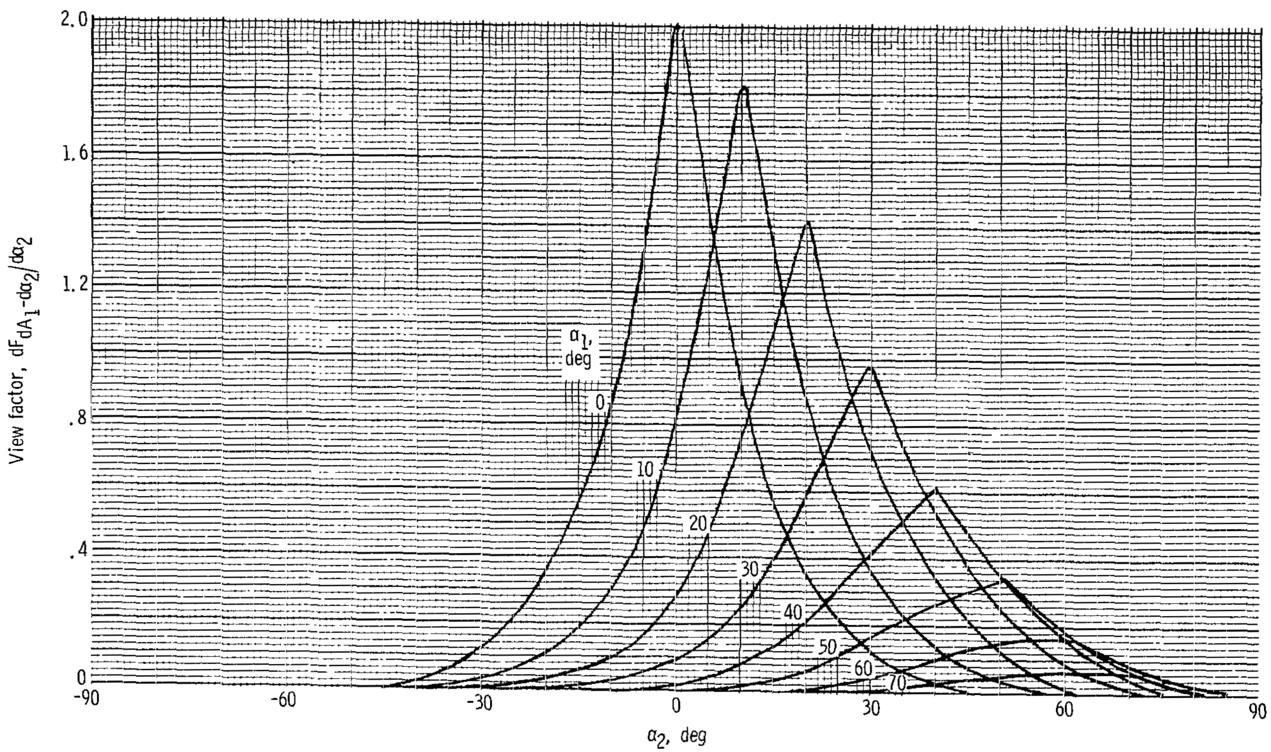
(g) $\rho = 0.60$.

Figure 5. - Continued.



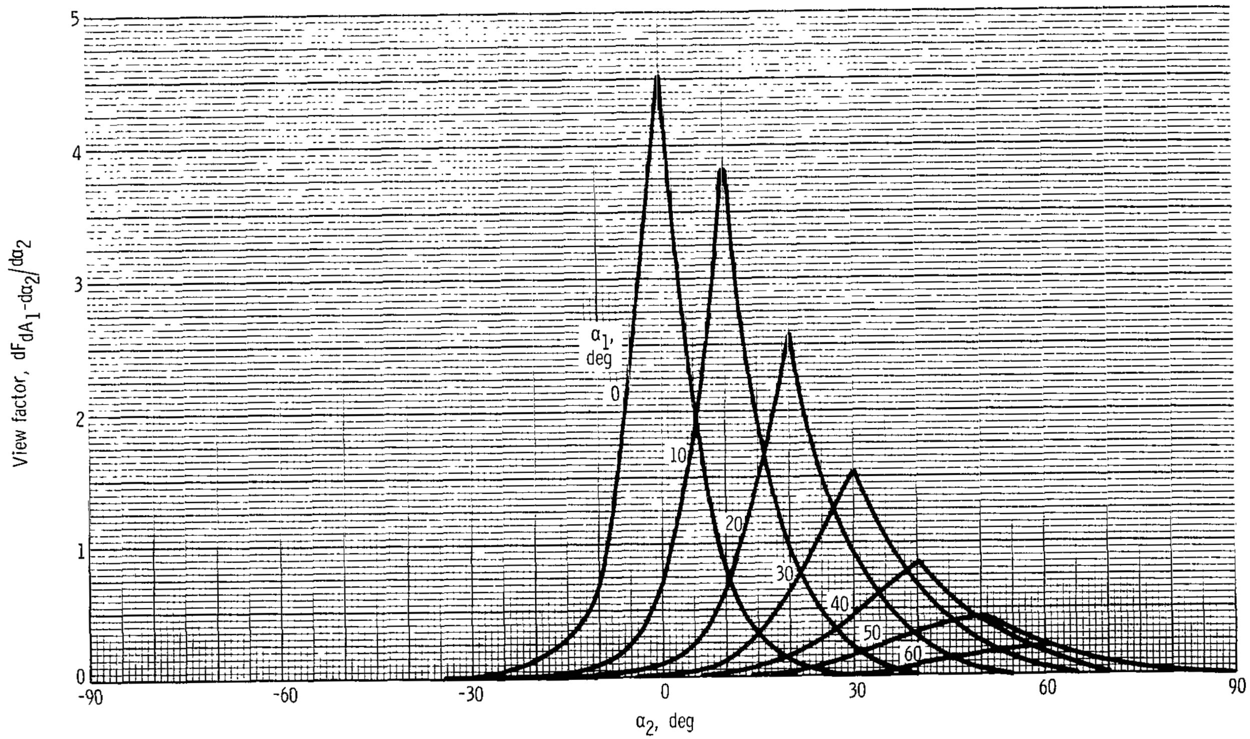
(h) $\rho = 0.70$.

Figure 5. - Continued.



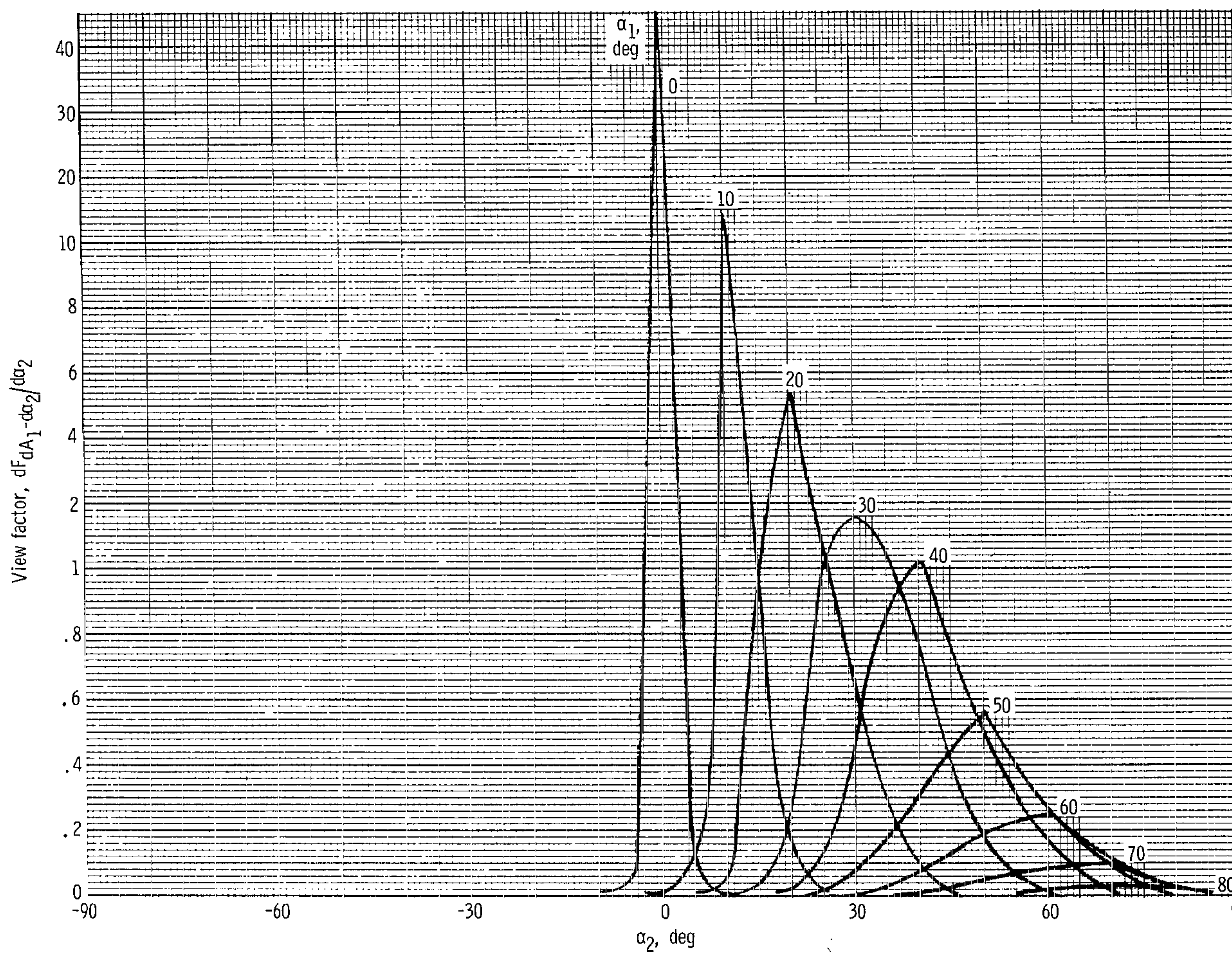
(i) $\rho = 0.80$.

Figure 5. - Continued.



(j) $\rho = 0.90$.

Figure 5. - Continued.



(k) $\rho = 0.99$.

Figure 5. - Continued.

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